***Example* 42.** Two industries input-output relationships are given below in **A** with final demand vector **B** (in units) :

$$\mathbf{A} = \begin{matrix} & I & II \\ I & 50 & 75 \\ II & 100 & 50 \end{matrix} \qquad \mathbf{B} = \begin{matrix} I & 75 \\ II & 50 \end{matrix}$$

$$\mathbf{A} = \begin{bmatrix} 50 & 75 \\ 100 & 50 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 75 \\ 50 \end{bmatrix}$$

If the gross output increases to $\begin{matrix} I \\ II \end{matrix}\begin{bmatrix} 400 \\ 600 \end{bmatrix}$, determine the final demand which can be satisfied.

Also test the Hawkins-Simon conditions. [*Delhi Univ. B.Com. (H) 1998*]

Solution. The inter-relationship between the production of two industries can be described in the following input-output table :

Producing Industry	*Input to Industry* I	II	*Final Demand*	*Total Output*
I	50	75	75	200
II	100	50	50	200

Let **R** denote the input-output coefficient matrix. Then

$$\mathbf{R} = \begin{bmatrix} 50/200 & 75/200 \\ 100/200 & 50/200 \end{bmatrix} = \begin{bmatrix} 1/4 & 3/8 \\ 1/2 & 1/4 \end{bmatrix}$$

Let $\mathbf{D} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$ denote the final demand which can be satisfied, if the gross output increases to

$\mathbf{X} = \begin{bmatrix} 400 \\ 600 \end{bmatrix}$. Since production equals consumption, we have

$$\mathbf{RX} + \mathbf{D} = \mathbf{X} \quad \Rightarrow \quad \mathbf{D} = \mathbf{X} - \mathbf{RX} = (\mathbf{I} - \mathbf{R})\mathbf{X}$$

Now $\quad \mathbf{I} - \mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/4 & 3/8 \\ 1/2 & 1/4 \end{bmatrix} = \begin{bmatrix} 3/4 & -3/8 \\ -1/2 & 3/4 \end{bmatrix}$

$\therefore \quad \mathbf{D} = (\mathbf{I} - \mathbf{R})\mathbf{X} = \begin{bmatrix} 3/4 & -3/8 \\ -1/2 & 3/4 \end{bmatrix}\begin{bmatrix} 400 \\ 600 \end{bmatrix} = \begin{bmatrix} 75 \\ 250 \end{bmatrix}$

To test Hawkins-Simon conditions, we have $|\mathbf{I} - \mathbf{R}| = 3/8 > 0$. Further each entry on the main diagonal of the input-output matrix **R** is less than 1. Hence Hawkins-Simon conditions are satisfied.

***Example* 43.** For a two sector economy, the input-output coefficient matrix is : $\mathbf{A} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.4 \end{bmatrix}$

If the final demands of the two sectors are 10 and 30, find the gross output. Land and labour are used as two primary inputs. Their coefficients for the two sectors are given as

$$\begin{matrix} \textit{Labour} \\ \textit{Land} \end{matrix} \begin{bmatrix} 0.4 & 0.3 \\ 0.5 & 0.4 \end{bmatrix}$$

If the wage rate and rent are ₹ 40 and ₹ 100 respectively, find the equilibrium prices for the two sectors. **[*Delhi Univ. B.Com. (H) 2006, 2011, 2014*]**

Solution. Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be the gross output required to meet the final demand $D = \begin{bmatrix} 10 \\ 30 \end{bmatrix}$.

Since production equals consumption, we have

$$\mathbf{X} = \mathbf{AX} + \mathbf{D} \quad \text{or} \quad (\mathbf{I} - \mathbf{A})\mathbf{X} = \mathbf{D}$$

Now $\mathbf{I}-\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.3 \\ -0.2 & 0.6 \end{bmatrix}$ $\therefore |\mathbf{I}-\mathbf{A}| = 0.30 - 0.06 = 0.24$

Since $|\mathbf{I}-\mathbf{A}| \neq 0$, the output $\mathbf{X}$ is given by $\mathbf{X} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{D}$. It can be easily verified that

$$(\mathbf{I}-\mathbf{A})^{-1} = \frac{1}{0.24}\begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.5 \end{bmatrix}$$

$$\therefore \quad \mathbf{X} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{D} = \frac{1}{0.24}\begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.5 \end{bmatrix}\begin{bmatrix} 10 \\ 30 \end{bmatrix} = \frac{1}{0.24}\begin{bmatrix} 15 \\ 17 \end{bmatrix} = \begin{bmatrix} 62.5 \\ 70.83 \end{bmatrix}$$

Thus the gross output for the two sectors are 62.5 and 70.83 respectively.

The cost of primary inputs for each unit of two sectors is given by

$$\begin{bmatrix} 40 & 100 \end{bmatrix}\begin{bmatrix} 0.4 & 0.3 \\ 0.5 & 0.4 \end{bmatrix} = \begin{bmatrix} 66 & 52 \end{bmatrix}$$

If p_1 and p_2 are the equilibrium prices for the two sectors, then

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \left[(\mathbf{I}-\mathbf{A})^{-1}\right]'\begin{bmatrix} 66 \\ 52 \end{bmatrix} = \frac{1}{0.24}\begin{bmatrix} 0.6 & 0.2 \\ 0.3 & 0.5 \end{bmatrix}\begin{bmatrix} 66 \\ 52 \end{bmatrix} = \frac{1}{0.24}\begin{bmatrix} 50 \\ 45.8 \end{bmatrix} = \begin{bmatrix} 208.33 \\ 190.83 \end{bmatrix}$$

Thus the equilibrium prices are: p_1 : ₹ 208.33 and p_2 : ₹190.83.