Example 8. If the supply function $x = f(p_1, p_2, ..., p_m)$ is homogeneous of degree n, show that sum of the partial price elasticities of supply equals n. (x denotes the quantity supplied of a particular commodity and $p_1, p_2, ..., p_m$ are the prices of the different commodities).

[*Delhi Univ. B.Com. (H) 1991, 1997*]

Solution. Let η_r denote the partial price elasticity of supply for x with respect to p_r ($r = 1, 2, ..., m$). Then

$$\eta_r = \frac{p_r}{x}\frac{\partial x}{\partial p_r}$$

Since $x = f(p_1, p_2, ..., p_m)$ is homogeneous of degree n, therefore by Euler's theorem, we have

$$p_1 \frac{\partial x}{\partial p_1} + p_2 \frac{\partial x}{\partial p_2} + ... + p_m \frac{\partial x}{\partial p_m} = nx$$

Dividing both sides of this equation by x, we obtain

$$\frac{p_1}{x}\frac{\partial x}{\partial p_1} + \frac{p_2}{x}\frac{\partial x}{\partial p_2} + ... + \frac{p_m}{x}\frac{\partial x}{\partial p_m} = n$$

i.e.,

$$\eta_1 + \eta_2 + ... + \eta_m = n$$

In other words, the sum of the partial price elasticities of supply equals n.

Example 9. Demand function for a commodity X is given by

$$D(x) = 300 - \frac{p_x^2}{2} + \frac{p_y}{50} + \frac{Y}{20},$$

where p_x is the price of the commodity, p_y is the price of a related commodity and Y the income of the consumer. Find the cross elasticity and income elasticity of demand for X when $p_x = 10$, $p_y = 15$ and $Y = 300$.

[*Delhi Univ. B.Com. (H) 1988*]

Solution. The cross partial elasticity of demand for X with respect to p_y is given by

$$\eta_{x,p_y} = \frac{p_y}{D(x)}\frac{\partial D(x)}{\partial p_y} = \frac{p_y}{300 - \frac{p_x^2}{2} + \frac{p_y}{50} + \frac{Y}{20}} \times \frac{1}{50}$$

Thus evaluating when $p_x = 10$, $p_y = 15$ and $Y = 300$, we obtain $\eta_{x,p_y} = \frac{3}{2653}$.

Income elasticity of demand for X is given by

$$\eta_{x,Y} = \frac{Y}{D(x)} \frac{\partial D(x)}{\partial Y} = \frac{Y}{300 - \frac{p_x^2}{2} + \frac{p_y}{50} + \frac{Y}{20}} \times \frac{1}{20}$$

Thus evaluating when $p_x = 10$, $p_y = 15$ and $Y = 300$, we obtain $\eta_{x,Y} = \frac{150}{2653}$.

Example 10. Find the partial elasticities of $z = x^2 e^y$.

Solution. Partial elasticity of z *w.r.t.* x is : $\eta_{z,x} = \frac{x}{z}\frac{\partial z}{\partial x} = \frac{x}{x^2 e^y} 2x\, e^y = 2$

Partial elasticity of z *w. r. t.* y is : $\eta_{z,y} = \frac{y}{z}\frac{\partial z}{\partial y} = \frac{y}{x^2 e^y} x^2\, e^y = y.$

EXERCISE 9.1