Symmetry between Primal and its Dual. It may be noted that there is a sort of symmetry between primal and its corresponding dual. Some of the symmetrical features can be stated as follows :

1. *If the primal is a maximization problem, then its dual is a minimization problem and vice versa.*

2. *If all constraints in the primal involve* $\leq$ ($\geq$)*, then all the constraints in its dual involve* $\geq$ ($\leq$).

3. *The coefficients in the dual's objective function are the constant terms in the primal's constraints and vice versa.*

4. *The number of structural variables in the dual problem is the number of constraints in the primal and vice versa.*

5. *The variables of the dual problem are different from the variables of its primal.*

6. *Dual of the dual is the primal itself.*

7. *The coefficient matrix of the left sides of the dual's constraints is the transpose of the coefficient matrix of the left sides of the primal's constraints.*

***Example* 32.** Write the dual to the following *LPP* :

Maximize $Z = 20x_1 + 15x_2 + 18x_3 + 10x_4$

subject to the constraints:

$$\begin{aligned} 4x_1 - 3x_2 + 10x_3 + 4x_4 &\le 60 \\ x_1 + x_2 + x_3 &= 27 \\ -x_2 + 4x_3 + 7x_4 &\ge 35 \\ x_1, x_2, x_3 &\ge 0; \quad x_4 : \text{unrestricted in sign} \end{aligned}$$

[*Delhi Univ. B.Com. (H) 2000*]

Solution. Since the primal is a maximization problem, we must convert each constraint into the type '$\le$'. Thus multiplying both sides of the third constraint by -1, we can restate the primal as

Maximize $Z = 20x_1 + 15x_2 + 18x_3 + 10x_4$

subject to the constraints:

$$\begin{aligned} 4x_1 - 3x_2 + 10x_3 + 4x_4 &\le 60 \\ x_1 + x_2 + x_3 + 0x_4 &= 27 \\ 0x_1 + x_2 - 4x_3 - 7x_4 &\le -35 \\ x_1, x_2, x_3 &\ge 0; \quad x_4 : \text{unrestricted in sign} \end{aligned}$$

Let y_1, y_2 and y_3 be the dual variables corresponding to three primal constraints in given order. Since the second constraint of the primal is an equation, the second variable y_2 will be unrestricted in sign. Similarly, the fourth variable x_4 of the primal problem is unrestricted in sign, therefore the fourth constraint of the dual will be an equation.

Hence the dual of the given problem is :

Minimize $Z^* = 60y_1 + 27y_2 - 35y_3$

subject to the constraints:

$$\begin{aligned} 4y_1 + y_2 &\ge 20 \\ -3y_1 + y_2 + y_3 &\ge 15 \\ 10y_1 + y_2 - 4y_3 &\ge 18 \\ 4y_1 - 7y_3 &= 10 \\ y_1, y_3 &\ge 0; \quad y_2 : \text{unrestricted in sign} \end{aligned}$$

Example 33. Find the dual of each of the following problems :

(*i*) Maximize $Z = 3x_1 + 5x_2 + 7x_3$

subject to the constraints:

$$\begin{aligned} x_1 + x_2 + 3x_3 &\le 10 \\ 4x_1 - x_2 + 2x_3 &\ge 15 \\ x_1, x_2 &\ge 0; \quad x_3 : \text{unrestricted in sign} \end{aligned}$$

[*Delhi Univ. B.Com. (H) 2011*]

(ii) Maximize $Z = 7x_1 + 8x_2 + 6x_3$
subject to the constraints:

$$x_1 + 4x_2 + 4x_3 = 8$$
$$3x_1 + 5x_2 + 3x_3 \leq 12$$
$$x_1, x_2, x_3 \geq 0$$

[*Delhi Univ. B.Com. (H) 2012*]

Solution. (*i*) To write its dual, we multiply both sides of second constraint by −1, so that the primal can be written in the form:

Maximize $Z = 3x_1 + 5x_2 + 7x_3$
subject to the constraints:

$$x_1 + x_2 + 3x_3 \leq 10$$
$$-4x_1 + x_2 - 2x_3 \leq -15$$
$$x_1, x_2 \geq 0; \quad x_3 : \text{unrestricted in sign}$$

Thus the dual is :

Minimize $Z^* = 10y_1 - 15y_2$
subject to the constraints:

$$y_1 - 4y_2 \geq 3$$
$$y_1 + y_2 \geq 5$$
$$3y_1 - 2y_2 = 7$$
$$y_1, y_2 \geq 0$$

(ii) The dual of the given problem is :

Minimize $Z^* = 8y_1 + 12y_2$
subject to the constraints:

$$y_1 + 3y_2 \geq 7$$
$$4y_1 + 5y_2 \geq 8$$
$$4y_1 + 3y_2 \geq 6$$
$$y_2 \geq 0; \quad y_1 : \text{unrestricted in sign}$$