Lecture 2

Covariance is the statistical measure that indicates the interactive risk of a security relative to others in a portfolio of securities. In other words, the way security returns vary with each other affects the overall risk of the portfolio.

The covariance between two securities X and Y may be calculated using the following
formula:

$$Cov_{xy} = \frac{[Rx - Rx][Ry - Ry]}{N}$$

Where:

Covxy = Covariance between x and y.

R_x = Return of security x.

R_y = Return of security y

R_x = Expected or mean return of security x.

R_y = Expected or mean return of security y.

N = Number of observations.

Calculation of Covariance

Year	**Rx**	**Deviation Rx – Rx**	**R…3 y**	**Deviation Ry – Ry**	**Product of deviations (Rx – Rx) (Ry – Ry)**
1	10	-4	17	5	-20
2	12	-2	13	1	-2
3	16	2	10	-2	-4
4	18	4	8	-4	-16

$$Cov_{xy} = \sum_{i=1}^{n} \frac{[Rx - Rx][Ry - Ry]}{N}$$

= -42 / 4 = -10.5

The covariance is a measure of how returns of two securities move together. If the returns of the two securities move in the same direction consistently the covariance would be positive. If the returns of the two securities move in opposite direction consistently the covariance would be negative. If the movements of returns are independent of each other, covariance would be close to zero.

Covariance is an absolute measure of interactive risk between two securities. To facilitate comparison, covariance can be standardized. Dividing the covariance between two securities by product of the standard deviation of each security gives such a standardised measure. This measure is called the coefficient of correlation. This may be expressed as:

$$r_{xy} = \frac{Cov_{xy}}{\sigma_x \sigma_y}$$

Where

R_{xy} = Coefficient of correlation between x and y

Co_{vxy} = Covariance between x and y.

σ_x = Standard deviation of x.

σy = Standard deviation of y

It may be noted from the above formula that covariance may be expressed as the product of correlation between the securities and the standard deviation of each of the securities. Thus,

$$Cov_{xy} = r_{xy}\sigma_x\sigma_y$$

The correlation coefficients may range from - 1 to 1. A value of -1 indicates perfect negative correlation between security returns, while a value of +1 indicates a perfect positive correlation. A value close to zero would indicate that the returns are independent.

The variance (or risk) of a portfolio is not simply a weighted average of the variances of the individual securities in the portfolio. The relationship between each security in the portfolio with every other security as measured by the covariance of return has also to be considered. Thus the variance of a portfolio with only two securities in it may be calculated with the following formula (already explained above)

$$\sigma^2_p = x_1{}^2\sigma_1{}^2 + x_2{}^2\sigma_2{}^2 + 2x_1x_2(r_{12}\sigma_1\sigma_2)$$

Where

σ_{2p}= Portfolio variance.

x_1 = Proportion of funds invested in the first security.

x_2 = Proportion of funds invested in the second security.

$\sigma_1{}^2$ = Variance of first security.

$\sigma_2{}^2$ = Variance of second security.

σ_1 = Standard deviation of first security

σ_2 = Standard deviation of second security.

r_{12} = Correlation coefficient between the returns of first and second security.

Portfolio standard deviation can be obtained by taking the square root of portfolio variance.

Let us take an example to understand the calculation of portfolio variance and portfolio standard deviation. Two securities P and Q generate the following sets of expected returns, standard deviations and correlation coefficient:

P	**Q**
r = 15 percent	20 percent
σ = 50 percent	30 percent
r_{pq}= -0.60	

A portfolio is constructed with 40 per cent of funds invested in P and the remaining 60 per cent of funds in Q.

The expected return of the portfolio is given by:

$$r_p = \sum_{i=1}^{n} x_i r_i$$

= (0.40 x 15) + (0.60 x 20) = 18 percent

The variance of the portfolio is given by:

$\sigma_{2p} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2(r_{12}\sigma_1\sigma_2)$

$= (0.40)^2\ (50)^2 + (0.60)^2\ (30)^2 + 2(0.40)\ (0.60)\ (-\ 0.60)\ (50)(30)$

= 400 + 324 - 432 = 292

The standard deviation of the portfolio is:

sp= $\sqrt{292}$ = 17.09 per cent.

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