Lecture 3

The return and risk of a portfolio depends on two sets of factors (a) the returns and risks of individual securities and the covariance between securities in the portfolio, (b) the proportion of investment in each security.

The first set of factors is parametric to the investor in the sense that he has no control over the returns, risks and covariances of individual securities. The second sets of factors are choice variables in the sense that the investor can choose the proportions of each security in the portfolio.

REDUCTION OF PORTFOLIO RISK THROUGH DIVERSIFICATION

The process of combining securities in a portfolio is known as diversification. The aim of diversification is to reduce total risk without sacrificing portfolio return. In the example considered above, diversification has helped to reduce risk. The portfolio standard deviation of 17.09 is lower than the standard deviation of either of the two securities taken separately, which were 50 and 30 respectively.

To understand the mechanism and power of diversification, it is necessary to consider the impact of covariance or correlation on portfolio risk more closely. We shall examine three cases: (a) when security returns are perfectly positively correlated, (b) when security returns are perfectly negatively correlated, and (c) when security returns are not correlated.

1. Security Returns Perfectly Positively Correlated

When security returns are perfectly positively correlated the correlation coefficient between the two securities will be +1. The returns of the two securities then move up or down together.

The portfolio variance is calculated using the formula:

$$\sigma^2 p = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2(r_{12}\sigma_1\sigma_2)$$

Since $r_{12} = 1$, this may be rewritten as:

$$\sigma^2_p = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2$$

The right hand side of the equation has the same form as the expansion of the identity $(a + b)^2$, namely $a^2 + 2ab + b^2$. Hence, it may be reduced as

$$\sigma^2 p = (x_1\sigma_1 + x_2\sigma_2)^2$$

The standard 1deviation then becomes

$\sigma_p = x_1\sigma_1 + x_2\sigma_2$

This is simply the weighted average of the standard deviations of the individual securities.

Taking the same example that we considered earlier for calculating portfolio variance, we shall calculate the portfolio standard deviation when correlation coefficient is +1.

Standard deviation of security P = 50

Standard deviation of security Q = 30

Proportion of investment in P = 0.4

Proportion of investment in Q = 0.6

Correlation coefficient = +1.0

Portfolio standard deviation may be calculated as:

$\sigma_p = x_1\sigma_1 + x_2\sigma_2$

= (0.4) (50) + (0.6) (30)

= 38

Being the weighted average of the standard deviations of individual securities, the portfolio standard deviation will lie between the standard deviations of the two individual securities. In our example, it will vary between 50 and 30 as the proportion of investment in each security changes.

For example, if the proportion of investment in P and Q are 0.75 and 0.25 respectively,

portfolio standard deviation becomes:

σ_p = (0.75) (50) + (0.25) (30) = 45

Thus, when the security returns are perfectly positively correlated, diversification provides only risk averaging and no risk reduction because the portfolio risk cannot be reduced below the individual security risk. Hence, diversification is not a productive activity when security returns are perfectly positively correlated.

2. Security Returns Perfectly Negatively Correlated

When security returns are perfectly negatively correlated, the correlation coefficient between them becomes -1. The two returns always move in exactly opposite directions.

The portfolio variance may be calculated as:

$\sigma^2 p = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2(r_{12}\sigma_1\sigma_2)$

Since r_{12} = -1, this may be rewritten as:

$\sigma^2 p = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 - 2x_1x_2(\sigma_1\sigma_2)$

The right hand side of the equation has the same form as the expansion of the identity (a - b) 2, namely a2 - 2ab + b2. Hence, it may be reduced as:

$\sigma_2 p = (x_1\sigma_1 - x_2\sigma_2)^2$

The standard deviation then becomes:

$\sigma_p = x_1\sigma_1 - x_2\sigma_2$

For the illustrative portfolio considered above, we can calculate the portfolio standard deviation when the correlation coefficient is —1.

$\sigma_p = (0.4)(50) - (0.6)(30) = 2$

The portfolio risk is very low. It may even be reduced to zero. For example, if the proportion of investment in P and Q are 0.375 and 0.625 respectively, portfolio standard deviation becomes:

$\sigma_p = (0.375)(50) - (0.625)(30) = 0$

Here, although the portfolio contains two risky assets, the portfolio has no risk at all. Thus, the portfolio may become entirely risk free when security returns are perfectly negatively correlated. Hence, diversification becomes a highly productive activity when securities are perfectly negatively correlated, because portfolio risk can be considerably reduced and sometimes even eliminated. But, in reality, it is rare to find securities that are perfectly negatively correlated.

3. Security Returns Uncorrelated

When the returns of two securities are entirely uncorrelated, the correlation coefficient would be zero.

The formula for portfolio variance is:

$\sigma^2_p = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x^1x^2(r_{12}\sigma_1\sigma_2)$

Since r_{12} = 0, the last term in the equation becomes zero; the formula may be rewritten

$\sigma^2_p = x_1^2 s_1^2 + x_2^2\sigma_2^2$

The standard deviation then becomes:

$\sigma_p = \sqrt{}x_1\sigma_1 + x_2\sigma_2$

For the illustrative portfolio considered above the standard deviation can be calculated when the correction coefficient is zero.

$\sigma_p = \sqrt{(0.4)^2(50)^2+(.6)^2(30)^2}$

$= \sqrt{400+324}$

$= 26.91$

The portfolio standard deviation is less than the standard deviations of individual securities in the portfolio. Thus, when security returns are uncorrelated, diversification reduces risk and is a productive activity.We may now tabulate the portfolio standard deviations of our illustrative portfolio having two securities P and Q, for different values of correlation coefficients between them. The proportion of investments in P and Q are 0.4 and 0.6 respectively. The individual standard deviations of P and Q are 50 and 30 respectively.

Portfolio Standard Deviations

Correlation Portfolio coefficients	**Correlation Portfolio standard deviations**
1.0	38.00
0.6	34.00
0.0	26.91
-0.6	17.09
-1.0	2.00

From the above analysis we may conclude that diversification reduces risk in all cases except when the security returns are perfectly positively correlated. As correlation coefficient declines from +1 to -1, the portfolio standard deviation also declines. But the risk reduction is greater when the security returns are negatively correlated.

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