Lecture 4

Example: 1

Calculate the expected return and variance of a portfolio comprising two securities, assuming that the portfolio weights are 0.75 for security 1 and 0.25 for security 2. The expected return for security 1 is 18 per cent and its standard deviation is 12 per cent, while the expected return and standard deviation for security 2 are 22 per cent and 20 per cent respectively. The correlation between the two securities is 0.6

Solution

Calculation of expected return of portfolio:

$$\overline{r_p} = \sum_{i=1}^{n} x_i \overline{r_i}$$

$= (0.75 \times 18) + (0.25 \times 22)$

$= 13.5 + 55 = 19$ per cent.

Calculation of portfolio variance:

$$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2 (r_{12} \sigma_1 \sigma_2)$$

$= (0.75)^2 (12)^2 + (0.25)^2 (20)^2 + 2(0.75)(0.25)(0.6 \times 12 \times 20)$

$= 81 + 25 + 54 = 160$ per cent.

Example: 2

Consider two securities, P and Q, with expected returns of 15 per cent and 24 per cent respectively, and standard deviation of 35 per cent and 52 per cent respectively. Calculate the standard deviation of a portfolio weighted equally between the two securities in their correlation is – 0.9.

Solution

$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1x_2 (r_{12} \sigma_1 \sigma_2)$

$= (0.5)^2 (35)^2 + (0.5)^2 (52)^2 + 2(0.5) (0.5) (-0.9 \times 35 \times 52)$

$\sigma = \sqrt{163.25} = 12.78$ per cent.

Example:3

The historical rates of return of two securities over the past ten years are given. Calculate the covariance and the correlation of the two securities.

Years	1	2	3	4	5	6	7	8	9	10
Security 1 (return per cent)	12	18	7	14	16	15	18	20	16	22
Security 2 (return per cent)	20	22	24	18	15	20	24	25	22	20

Solution

Year	**R1**	**Deviation ($R_1 - \overline{R}_1$)**	**R2**	**Deviation ($R_2 - \overline{R}_2$)**	**Product of deviations**
1	12	-2.8	20	-1	2.8
2	8	-6.8	22	1	-6.8
3	7	-7.8	24	3	-23.4
4	14	-0.8	18	-3	2.4
5	16	1.2	15	-6	-7.2
6	15	0.2	20	-1	-0.2
7	18	3.2	24	3	9.6
8	20	5.2	25	4	20.8
9	16	1.2	22	1	1.2
10	22	7.2	20	-1	-7.2
$R_1 = 148/10 = 14.8$			**$R_2 = 210/10 = 21$**		**-8.00**

Covariance = -8/ 10 = -0.8

For calculation of correlation, the standard deviation of the two securities are also required.

Calculation of Standard Deviation

Year	R_1	R_1^2	R_2	$R2^2$
1	12	144	20	400
2	8	64	22	484
3	7	49	24	576
4	14	196	18	324
5	16	256	15	225
6	15	225	20	400
7	18	324	24	576
8	20	400	25	625
9	16	256	22	484
10	22	484	20	400
	148	**2398**	**210**	**4494**

Standard deviation of security 1:

$$\sigma_1 = \sqrt{\frac{N \sum R_1^2 - (\sum R_1)^2}{N^2}}$$

$$= \sqrt{\frac{(10 \times 2398) - (148)^2}{10 \times 10}}$$

$$= \sqrt{\frac{23980 - 21904}{100}}$$

$$= \sqrt{20.76}$$

$$= 4.56$$

Standard deviation of security 2:

$$\sigma_1 = \sqrt{\frac{N \sum R_2^2 - (\sum R_2)^2}{N^2}}$$

$$= \sqrt{\frac{(10 \times 4494) - (210)^2}{10 \times 10}}$$

$$= \sqrt{\frac{44940 - 44100}{100}}$$

$$= \sqrt{840}$$

$$= 28.98$$

Correlation:

$$r_{12} = \frac{Cov_{12}}{\sigma_1 \sigma_2}$$

$$= \frac{-0.8}{4.56 \times 28.98}$$

$$= \frac{-0.8}{132.15}$$

$$= -0.0061$$

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