Lecture 5

Portfolio Risk

In order estimate the total risk of a portfolio of assets, several estimates are needed: the variance of each individual asset under consideration for inclusion in the portfolio and the covariance, or correlation co-efficient, of each asset with each of the other assets.

Table shows the returns on two securities and on a portfolio that includes both of them. Security X constitutes 60 per cent of the market value of the portfolio and security Y the other 40 per. The predicted return on the portfolio is simply a weighted average of the predicted returns on the securities, using the proportionate values as weights. Summary measures show values computed from the estimates in Table. The expected return for the portfolio is simply the weighted average of the expected returns on its securities, using the proportionate values as weights (17.0% = 6 x 15% + 4 x 20%). However, this is not true for either the variance or the standard deviation of return for the portfolio smaller than the corresponding values for either of the component securities. This rather surprising result has a simple explanation. The risk of a portfolio depends not only on the risk of its securities, considered in isolation, but also on the extent to which they are affected similarly by underlying events. To illustrate this, two extreme cases are shown in Table . In the first case both

TABLE

Portfolio and Security Risks

A. RETURN

Even t	Probability	Return on Security X	Return on Security Y	Return on Portfolio
(1)	(2)	(3)	(4)	(5) = 6 x (3) + 4 x (4)
a	.20	-10%	5.0%	-4.0%
b	.40	25	30.0	27.0
c	.30	20	20.0	20.0
d	.10	10	10.0	10.0

B. SUMMARY MEASURES

	Security X	Security Y	Portfolio
Expected Return	15.0	20.0	17.0
Variance of Return	175.0	95.0	135.8
Standard deviation of Return	13.2287	9.7468	11.65

C. COVARIANCE AND CORRELATIONS

Even t	Probabilit y	Deviation of Return for Security X	Deviation of Return for Security Y	Product of Deviation	Probability Times Product of Deviation
(1)	(2)	(3)	(4)	(5) = (3) x (4)	(6) = (2) x (5)
a	.20	-25.0%	-15.0%	375	75.00
b	.40	10.0	10.0	100	40.00
c	.30	5.0	0	0	0
d	.10	-5.0	-10.0	50	5.00
					Covariance = 120

$$\text{Correlation co-efficient} = \frac{120.00}{13.2287 \times 9.7468} = 0.9307$$

The variance and the standard deviation of the portfolio are the same as the corresponding values for the securities. Then diversification has no effect at all on risk. In the second case the situation is very different. Here the security’s returns offset one another in such a manner that the particular combination that makes up this portfolio has no risk at all. Diversification has completely eliminated risk. The difference between these two cases concerns the extent to which the security’s returns are correlated i.e., tend to “to-together”. Either of two measures can be used to state the degree of such a relationship: the covariance or the correlation co-efficient.

TABLE

Risk and Return for a Two-Security Portfolio

A. TWO SECURITIES WITH EQUAL RETURNS

Even t	Probability	Return on Security X %	Return on Security Y %	Return on Portfolio
(1)	(2)	(3)	(4)	(5) = 6 x (3) + 4 x (4)
A	.20	-10.0	-10.0	-10.0
B	.40	25.0	25.0	25.0
C	.30	20.0	20.0	20.0
D	.10	10.0	10.0	10.0
Expected Return		15.0	15.0	15.0
Variance of Return		175.0	175.0	175.0
Standard deviation of Return		13.2287	13.2287	13.2287

B. TWO SECURITIES WITH OFFSETING RETURNS

Even t	Probability	Return on Security X %	Return on Security Y %	Return on Portfolio
(1)	(2)	(3)	(4)	(5)
A	.20	-10.	40.0	10.0
B	.40	25.0	-20.0	10.0
C	.30	20.0	-5.0	10.0
D	.10	10.0	10.0	10.0
Expected Return (%)		15.0	-0.5	10.0
Variance of Return		175.0	37.47	0
Standard deviation		13.228	6.1217	0

The computations required to obtain the covariance for the two securities are presented in Table 3C. The deviation of each security's return from its expected value is determined and the product of the two obtained (column 5). The variance is simply a weighted average of such products, using the probabilities of the events as weights. A positive value for the covariance indicates that the securities returns tend to go together – for example, a better-that-expected return for one is likely to occur along with a better-than-expected return for the other. A small or zero value for the covariance indicates that there is little or no relationship between the two returns. The correlation coefficient is obtained by dividing the covariance by the product of the two security's standard deviation. As shown in Table – 3C, in this case the value is 0.9307.

Correlation coefficients always lie between +1.0 and –1.0, inclusive. The former value represents perfect positive correlation, of the type shown in the example in Table – 4A. The latter value represents perfect negative correlation in Table – 4B. The relationship between the covariance and the correlation coefficient can be represented as follows:

$$C_{XY} = R_{XY} S_X S_Y \qquad (1)$$

or

$$R_{XY} = \frac{C_{XY}}{S_X S_Y} \qquad (2)$$

where :

C_{XY} = covariance between return on X and return on Y.

r_{XY} = coefficient of correlation between return on X and return on Y.

S_X = standard deviation of return on X.

S_Y = standard deviation of return on Y.

For two securities, X and Y, the relationship between the risk of a portfolio of two securities and the relevant variables, the formula is:

$$V_P = W^2_X V_X + 2W_X W_Y C_{XY} + W^2_Y V_Y \qquad (3)$$

where :

V_P = the variance of return for the portfolio.

V_X = the variance of return for the security X.

V_Y = the variance of return for the security Y.

C_{XY} = the covariance between the return on security X and the return On security Y.

W_X = the proportion of the portfolio's value invested in security X.

W_Y = the proportion of the portfolio's value invested in security Y.

For the case shown in Table-3

W_X = 0.6; W_Y = .4

V_X = 175.0 V_Y = 95.0 C_{XY} = 120.00

Inserting these values in formula (3), we get the variance of the portfolio as a whole:

$$V_P = (0.6)^2 \times 175.0 + 2 \times .6 \times .4 \times 120 + (0.4)^2 \times 95.0$$
$$= 63.00 + 57.60 + 15.20$$
$$= 135.80$$

The relationship that gives the variance for a portfolio with more than two securities is similar in nature but more extensive. Both the risks of the securities and all their correlations have to be taken into account. The formula is:

$$V_P = \sum_{x=1}^{N} \sum_{y=1}^{N} W_X W_Y C_{XY} \qquad (4)$$

$$= \sum_{x=1}^{N} \sum_{y=1}^{N} W_X W_Y r_{XY} \sigma_X \sigma_Y$$

where :

V_P = the variance of return for the portfolio.

W_X = the proportion of the portfolio's value invested in security X.

W_Y = the proportion of the portfolio's value invested in security Y.

C_{XY} = the covariance between the return on security X and the return On security Y.

N = the number of securities.

The two summation signs mean that every possible combination must be included in the total, with a value between 1 and N substituted where x appears and a value between 1 and N substituted where y appears. In those cases in which the values are the same, the relevant covariance is that between a security's return and itself.

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