Lecture 6: Portfolio Risk, r and Diversification Effects

Perfectly Positively Correlated Returns

The returns from two securities are perfectly positively correlated when a cross-plot gives points lying precisely on a upward-sloping straight line, as shown in Figure – 1A. Each point indicates the return on security A (horizontal axis) and the return on security B (vertical axis) corresponding to one event. The example shown in Table – 4A confirms to this pattern.

What is the effect on risk when two securities of this type are combined? The general formula is:

$$V_P = W^2_X V_X + 2W_X W_Y C_{XY} + W^2_Y V_Y$$

The covariance term can, of course, be replaced, using formula (1):

$$C_{XY} = R_{XY}\, SX\, S_Y$$

However, in this case there is perfect positive correlation, so $r_{XY} = +1$ and $C_{XY} = S_X S_Y$. As always, $V_X = S^2_X$, $V_Y = S^2_Y$ and $V_P = S^2_P$

Substituting all these values in general formula gives:

$$S^2_P = W^2_X S^2_X + 2W_X\ W_Y\ S_X\ S_Y + W^2_Y\ S^2_Y$$

$$S^2_P = (W_X S_X + W_Y S_Y)^2$$

$$S^2_P = W_X S_X + W_Y S_Y \quad \text{when} \quad r_{XY} = +1 \qquad (5)$$

This is an important result. When two securities returns are perfectly positively correlated, the risk of a combination, measured by the standard deviation of return, is just a weighted average of the risks of the component securities, using market value as weights. The principle holds as well if more than two securities are included in a portfolio. In such cases, diversification does not provide risk reduction but only risk averaging.

Perfectly Negatively Correlated Returns

Diversification can eliminate risk in case of perfectly negatively correlated returns. Since $r_{XY} = -1$, the general formula becomes:

$$S^2_P = W^2_X S^2_X - 2W_X W_Y S_X S_Y + W^2_Y S^2_Y$$

This can be factored to obtain:

$$S^2_P = (W_X S_X - W_Y S_Y)^2 \quad \text{when} \quad r_{XY} = -1 \qquad (6)$$

Assuming a portfolio, in which the proportionate holdings are inversely related to the relative risks of the two securities, i.e.:

$$\frac{W_X}{W_Y} = \frac{S_Y}{S_X} \quad \text{or } W_X = \frac{S_Y W_Y}{S_X}$$

For this combination the parenthesized term in formula (6) will be:

$$W_X S_X - W_Y S_Y = \frac{S_Y W_Y}{S_X} S_X - W_Y S_Y = 0$$

If this term is zero, of course, the portfolio's standard deviation of return must be zero as well. When two securities returns are perfectly negatively correlated, it is possible to combine them in a manner that will eliminate all risk. Figure – 1b shows the returns from two securities perfectly negatively correlated, a cross-plot gives points lying precisely on a downward-sloping straight line. The example shown in Table – 4b confirms to this pattern. This principle motivates all hedging strategies. This object is to take position that will offset each other with regard to certain kinds of risk, reducing or completely eliminating such sources of uncertainty.

Uncorrelated Returns

Some risks can be substantially reduced by pooling. This has crucial implications for investment management. Most importantly, it provides the basis for understanding the relationship between risk and return. A special case of extreme importance arises when a cross-plot of security returns shows no pattern that can be represented even approximately by an upward-sloping or downward-sloping line. (See Figure – 1c). In such an instance, the returns are uncorrelated. The correlation coefficient, , is zero, as is the covariance. In this situation, the general formula becomes:

$$S^2_P = W^2_X S^2_X + W^2_Y S^2_Y \quad \text{when} \quad r_{XY} = 0 \qquad (7)$$

To illustrate the diversification effect, consider a portfolio divided equally between two securities of equal risk, say 20.0%. That is:

$W_X = .5; \quad W_Y = .5; \quad S_X = 20; \quad S_Y = 20$

Substitution these values in equation (7) we get:

$$(.5)^2 (20)^2 + (.5)^2 (20)^2 = (.25)(400) + (.25)(400)$$

Thus:

$$S^2_P = 200 \text{ and } S_P = 14.14$$

Diversification has helped as the risk of the portfolio is less that the risk of either of its component securities. The result will remain same irrespective of the number of securities. However, when all returns are uncorrelated the complete formula becomes:

$S^2_P = W^2_1 S^2_1 + W^2_2 S^2_2 + \ldots\ldots\ldots + W^2_N S^2_N$

where:

S_P …… = The standard deviation of the return on portfolio.

W_1, W_2..= The proportions invested in securities 1,2, etc.

S_1, S_2 .. = The standard deviation of the returns for securities 1,2,etc.

N = The number of securities included.

This is an extremely important relationship for investment analysis and also provides the bases for insurance, or risk pooling. This can be seen by extending the previous example and assuming a portfolio of equal parts of a number of securities, each with a risk (standard deviation of return) of 20%. If two securities are included:

$$S^2_P = (1/2)^2\ 20^2 + (1/2)^2\ 20^2$$
$$= 2(1/2)^2\ 20^2$$

If three securities are included:

$$S^2_P = (1/3)^2\ 20^2 + (1/3)^2\ 20^2 + (1/3)^2\ 20^2 = 3(1/3)^2\ 20^2$$

To generalize, represent the number of securities by N. Then:

$$S^2_P = (1/N)^2\ 20^2 + (1/N)^2\ 20^2 + \ldots\ldots\ldots$$
$$= N(1/N)^2\ 20^2$$

Simplifying:

$$S^2_P = N/N^2\ 20^2 = 20^2/N$$
$$S_P = 20/\sqrt{N}$$

Diversification provides substantial risk reduction if the components of a portfolio are uncorrelated. In fact, if enough securities are included, the overall risk of the portfolio will be almost (but not quite) zero. This is why insurance companies attempt to write many individual policies and spread their coverage so as to minimize overall risk.

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