PORTFOLIO SELECTION:

The objective of every rational investor is to maximise his returns and minimise the risk. Diversification is the method adopted for reducing risk. It essentially results in the construction of portfolios. The proper goal of portfolio construction would be to generate a portfolio that provides the highest return and the lowest risk. Such a portfolio would be known as the optimal portfolio. The process of finding the optimal portfolio is described as ***portfolio selection***. The conceptual framework and analytical tools for determining the optimal portfolio in disciplined and objective manner have been provided by Harry Markowitz in his pioneering work on portfolio analysis described in 1952 Journal of Finance article and subsequent book in 1959. His method of portfolio selection has come to be known as the Markowitz model. In fact, Markowitz‘s work marks the beginning of what is known today as modern portfolio theory.

In finance, the **Markowitz model** - put forward by Harry Markowitz in 1952 - is a portfolio optimization model; it assists in the selection of the most efficient portfolio by analyzing various possible portfolios of the given securities. Here, by choosing securities that do not 'move' exactly together, the HM model shows investors how to reduce their risk. The HM model is also called mean-variance model due to the fact that it is based on expected returns (mean) and the standard deviation (variance) of the various portfolios. It is foundational to Modern portfolio theory.

Assumptions

Markowitz made the following assumptions while developing the HM model:

1. Risk of a portfolio is based on the variability of returns from the said portfolio.
2. An investor is risk averse.
3. An investor prefers to increase consumption.
4. The investor's utility function is concave and increasing, due to their risk aversion and consumption preference.
5. Analysis is based on single period model of investment.
6. An investor either maximizes their portfolio return for a *given* level of risk or maximizes their return for the *minimum* risk.
7. An investor is rational in nature.

To choose the best portfolio from a number of possible/Feasible set of portfolios, each with different return and risk, two separate decisions are to be made, detailed in the below sections:

1. Determination of a set of efficient portfolios.
2. Selection of the best portfolio out of the efficient set.

Feasible set of portfolios:

With a limited number of securities an investor can create a very large number of portfolios by combining these securities in different proportions. These constitute the feasible set of portfolios in which the investor can possibly invest. This is also known as the portfolio opportunity set.

Each portfolio in the opportunity set is characterized by an expected return and a measure of risk, viz., variance or standard deviation of returns. Not every portfolio in the portfolio opportunity set is of interest to an investor. In the opportunity set some portfolios will obviously be dominated by others. A portfolio will dominate another if it has either a lower standard deviation and the same expected return as the other, or a higher expected return and the same standard deviation as the other. Portfolios that are dominated by other portfolios are known as inefficient portfolios. An investor would not be interested in all the portfolios in the opportunity set. He would be interested only in the efficient portfolios.

Efficient set of portfolios:

Let us consider various combinations of securities and designate them as portfolios 1 to n. The expected returns of these portfolios may be worked out. The

risk of these portfolios may be estimated by measuring the standard deviation of portfolio returns. The table below shows illustrative figures for the expected returns and standard deviations of some portfolios.

Portfolio no.	***Expected Return (per cent)***	***Standard deviation (risk)***
1	5.6	4.5
2	7.8	5.8
3	9.2	7.6
4	10.5	8.1
5	11.7	8.1
6	12.4	9.3
7	13.5	9.5
8	13.5	11.3
9	15.7	12.7
10	16.8	12.9

If we compute portfolio nos. 4 and 5, for the same standard deviation of 8.1 portfolio no. 5 gives a higher expected return of 11.7, making it more efficient than portfolio no. 4.

Again, if we compare portfolio nos. 7 and 8, for the same expected return of 13.5 per cent, the standard deviation is lower for portfolio no. 7, making it more efficient than portfolio no. 8. Thus, the selection of portfolio by the investor will be guided by two criteria:

1. Given two portfolios with the same expected return, the investor would prefer the one with the lower risk.
2. Given two portfolios with the same risk, the investor would prefer the one with the higher expected return.

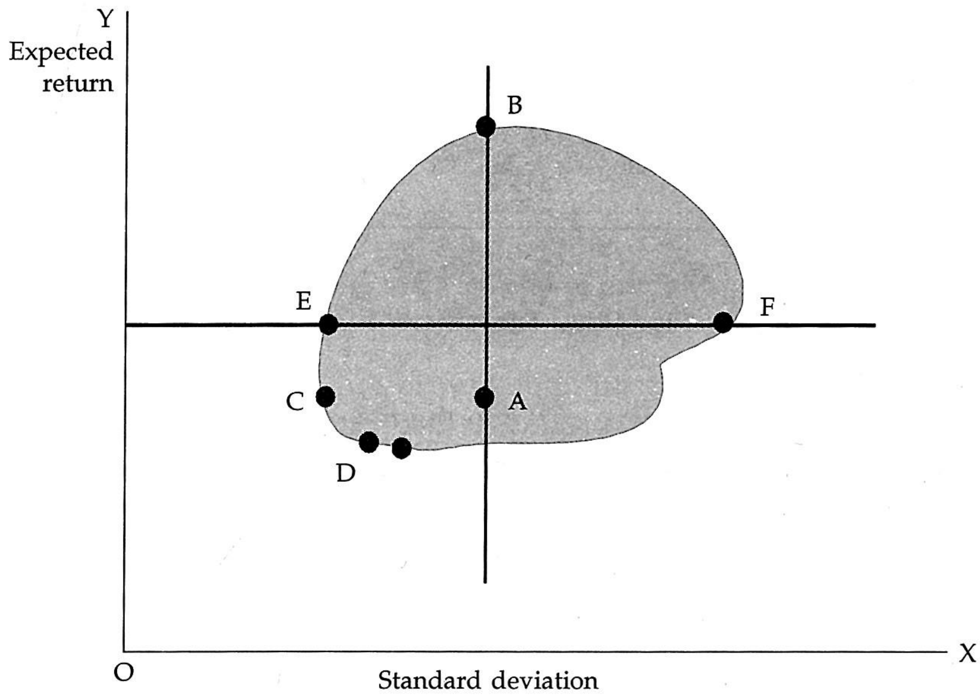
These criteria are based on the assumption that investors are rational and also risk-averse. As they are rational they would prefer more return to less return. As they are risk-averse, they would prefer less risk to more risk.

The concept of efficient sets can be illustrated with the help of a graph. The expected return and standard deviation of portfolios can be depicted on an XY graph, measuring the expected return on the Y axis and the standard deviation on the X axis. Following figure depicts such a graph.

As each possible portfolio in the opportunity set or feasible set of portfolios has an expected return and standard deviation associated with it, each portfolio would be represented by a single point in the risk-return space enclosed within the two axes of the graph. The shaded area in the graph represents the set of all possible portfolios that can be constructed from a given set of securities. This opportunity set of portfolios takes a concave shape because it consists of portfolios containing securities that are less than perfectly correlated with each other.

Diagram:

Consider portfolios F and E. Both the portfolios have the same expected return but portfolio E has less risk. Hence, portfolio E would be preferred to portfolio F. Now consider portfolios C and E. Both have the same risk, but portfolio E offers more return for the same risk. Hence, portfolio E would be preferred to portfolio C. Thus, for any point of risk-return space, an investor would like to move as far as possible in the direction of increasing returns and also as far as possible in the direction of decreasing risk. Effectively, he would be moving towards the left in search of decreasing risk and upwards in search of increasing returns.

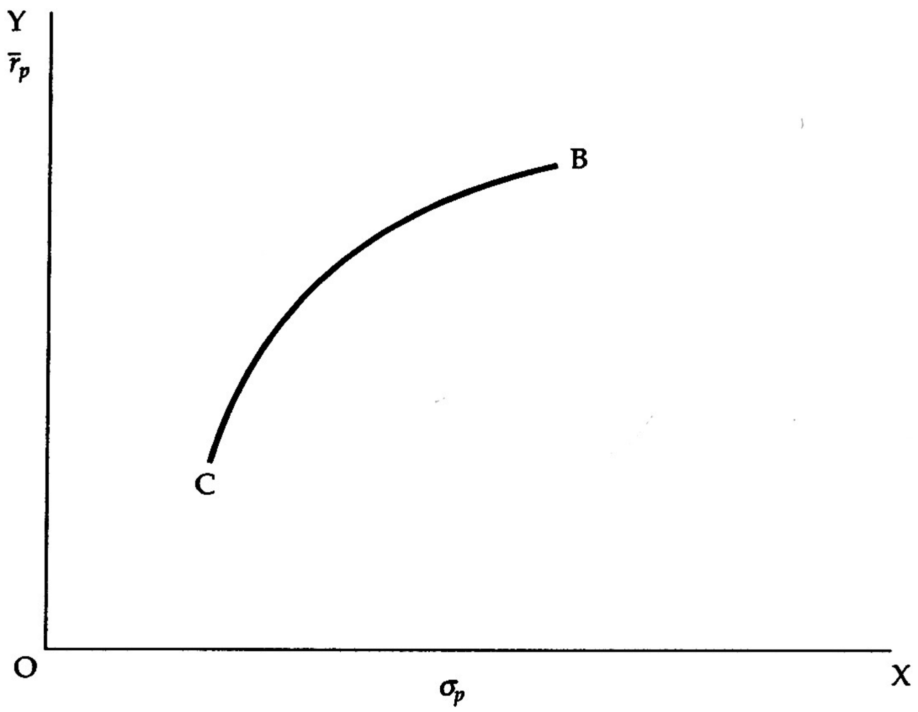


Feasible set of portfolios

Let us consider portfolios C and A. Portfolio C would be preferred to portfolio A because it offers less risk for the same level of return. In the opportunity set of portfolios represented in the diagram, portfolio C has the lowest risk compared to all other portfolios. Here portfolio C in this diagram represents the global minimum variance portfolio.

Comparing portfolios A and B, we find that portfolio B is preferable to portfolio A because it offers higher return for the same level of risk. In this diagram, point B represents the portfolio with the highest expected return among all the portfolios in the feasible set.

Thus, we find that portfolios lying in the north-west boundary of the shaded area are more efficient than all the portfolios in the interior of the shaded area. This boundary of the shaded area is called the efficient frontier because it contains all the efficient portfolios in the opportunity set. The set of portfolios lying between the global minimum variance portfolio and the maximum return portfolio on the efficient frontier represents the efficient set of portfolios. The efficient frontier is shown separately in the following figure:



The efficient frontier

The efficient frontier is a concave curve in the risk-return space that extends from the minimum variance portfolio to the maximum return portfolio.

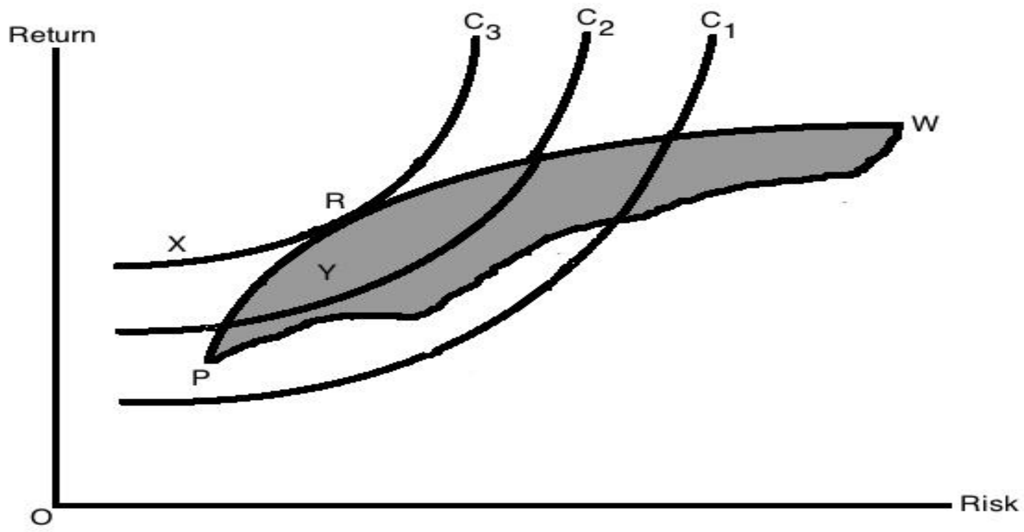
Selection of optimal portfolio:

The portfolio selection problem is really the process of delineating the efficient portfolios and then selecting the best portfolio from the set.

Rational investors will obviously prefer to invest in the efficient portfolios. The particular portfolio that an individual investor will select from the efficient frontier will depend on that investor‘s degree of aversion to risk. A highly risk averse investor will hold a portfolio on the lower left hand segment of the efficient frontier, while an investor who is not too risk averse will hold one on the upper portion of the efficient frontier.

The selection of the optimal portfolio thus depends on the investor‘s risk aversion, or conversely on his risk tolerance. This can be graphically represented through a series of risk return utility curves or indifference curves. The indifference curves of an investor are shown in the figure below. Each curve represents different combinations of risk and return all of which are equally satisfactory to the concerned investor. The investor is indifferent between the successive points in the curve. Each successive curve moving upwards to the left represents a higher level of satisfaction or utility. The investor‘s goal would be to maximise his utility by moving upto the higher utility curve. The optimal portfolio for an investor would be the one at the point of tangency between the efficient frontier and the risk-return utility or indifference curve.

This is shown in the following figure. The point R‘ represents the optimal portfolio.



Optimal portfolio

R is the point where the efficient frontier is tangent to indifference curve C_3, and is also an efficient portfolio. With this portfolio, the investor will get highest satisfaction as well as best risk-return combination (a portfolio that provides the highest possible return for a given amount of risk). Any other portfolio, say X, isn't the optimal portfolio even though it lies on the same indifference curve as it is outside the feasible portfolio available in the market. Portfolio Y is also not optimal as it does not lie on the best feasible indifference curve, even though it is a feasible market portfolio. Another investor having other sets of indifference curves might have some different portfolio as their best/optimal portfolio.

Limitations of Markowitz model:

1. ***Large number of input data required for calculations:*** An investor must obtain estimates of return and variance of returns for all securities as also covariances of returns for each pair of securities included in the portfolio. If there are *N* securities in the portfolio, he would need *N* return estimates, *N* variance estimates and *N* (*N*-1) / 2 covariance estimates, resulting in a total of 2*N* + [*N* (*N*-1) / 2] estimates. For example, analysing a set of 200 securities would require 200 return estimates, 200 variance estimates and

19,900 covariance estimates, adding upto a total of 20,300 estimates. For a set of 500 securities, the estimates would be 1,25,750. Thus, the number of estimates required becomes large because covariances between each pair of securities have to be estimated.

2. ***Complexity of computations required:*** The computations required are numerous and complex in nature. With a given set of securities infinite number of portfolios can be constructed. The expected returns and variances of returns for each possible portfolio have to be computed. The identification of efficient portfolios requires the use of quadratic programming which is a complex procedure.

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