Q1 Mr Ram is curious to know whether the following 5 stocks are appropriately valued in the market. Accordingly, he creates a table (shown below) listing the betas of each stock along with their ex-ante expected return values that have been calculated using a probability distribution. He also lists the current risk-free rate and the expected rate of return on the broad market index. Help him out and state your steps.

Stock	Expected Return	Beta
1	22%	1.8
2	8%	0.9
3	14%	1.2
4	10%	1.1
5	16%	1.4
R_f	3.5%	----
R_m	15%	1.0

ANSWER (a)

Step 1. Using the CAPM equation calculate the risk-based return of each stock

Stock

Stock	Expected Return	Beta	CAPM $E(R_i)$	Comment
1	26%	1.8	24.20%	Undervalued
2	16%	0.9	13.85%	Undervalued
3	14%	1.2	17.30%	Overvalued
4	16.15%	1.1	16.15%	Correctly valued
5	20%	1.4	19.60%	Undervalued
R_f	3.50%	----		
R_m	15%	1		

Step 2. If CAPM-based E(R) is less than the ex-ante return listed, the stock is undervalued, i.e. it is expected to earn a higher rate than it should, based on its beta.

Hence, Stocks 1, 2, and 5 are undervalued, while Stock3 is overvalued, and Stock 4 is correctly valued.

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b If Ram wants to form a 2-stock portfolio of the most undervalued stocks with a beta of 1.3, how much will she have to weight each of the stocks by?

ANSWER (b)

Based on the results in (A), Stocks 1 and 2 are most undervalued and would be chosen by Ram to form the 2-stock portfolio with a beta = 1.3.

Stock 1's beta = 1.8; Stock 2's beta = 0.9; Desired Portfolio beta = 1.3

Since the portfolio beta = weighted average of individual stock betas

Let Stock 1's Weight be X%; Thus Stock 2's Weight would be (1-X)%

→→ $1.8 \times X\% + 0.9 \times (1 - X)\% = 1.3$

→→ $1.8X + 0.9 - 0.9X = 1.3$

→→ 0.9X = 0.4

→→ X = 0.4/0.9 = 0.4444 or 44.44% = Stock 1's Weight

→→ (1 – X) = 1 – 0.4444 = .5556 or 55.56 = Stock 2's Weight

Check….0.4444 × 1.8 + 0.5556 × 0.9 = 0.79992+ 0.50004=1.3

Q2 Anita is curious to know what her portfolio's CAPM-based expected rate of return should be. After doing some research she figures out the market values and betas of each of her 5 stocks (shown below) and is told by her consultant that the risk-free rate is 3% and the market risk premium is 8%. Help Anita calculate her portfolio's expected rateof return.

ANSWER

Stock	Value	Weight	Beta
1	Rs35,000	0.1400	1.6
2	Rs40,000	0.1600	1.2
3	Rs45,000	0.1800	1.0
4	Rs50,000	0.2000	-0.8
5	Rs80,000	0.3200	0.8
	Rs250,000		

First determine the portfolio beta using the following formula:

$$\beta_p = \sum_{i=1}^{n} w_i \times \beta_i \qquad \textbf{8.10}$$

Portfolio Beta = 0.14 × 1.6+.16 × 1.2+.18 × 1.0+.2 × -0.8+.32 × 0.8
= 0.224 + 0.192 +0.18 + (-.16) + 0.256 =0.692

Next, using the CAPM equation and r_f = 3%, $E(r_m-r_f)$ = 8%; calculate ***the portfolio's expected rate.***

$E(r_p)$ = 3% + 8% × (.692) = 8.54%

Q3 Using the probability distribution shown below, calculate the expected risk and return estimates of each stock and of a portfolio comprised of 40% of Stock A and 60% of Stock B.

State of Economy	Probability of State occurring	Stock A's Conditional return	Stock B's Conditional return
Recession	0.3	-12%	20%
Normal	0.5	14%	12%
Boom	0.2	25%	-10%

ANSWER

Stock A's expected return = 0.3 × (-12%)+0.5 × (14%)+0.2 × (25%)=8.4%
Stock B's expected return = 0.3 × (20%)+0.5 × (12%)+0.2 × (-10%)=10%
Stock A's expected variance = $0.3 \times (-12-8.4)^2+0.5 \times (14-8.4)^2+0.2 \times (25-8.4)^2$
= 124.848 + 15.68 + 55.112
= 195.64

Stock A’s expected std. dev. = $\sqrt{195.64}$ = 13.99%

Stock B’s expected variance = $0.3 \times (20-10)^2+0.5 \times (12-10)^2+0.2 \times (-10-10)^2$

= 30 + 2 + 80

= 112

Stock B’s expected std. dev. = $\sqrt{112}$ = 10.58%

Portfolio AB’s expected return = Wt. in A × $E(R_A)$ + Wt. in B × $E(R_B)$

= .4 × 8.4% + .6 × 10% = 9.36%

OR

We can calculate the portfolio’s conditional returns and then compute the expected return and standard deviation/variance.

Portfolio AB’s recession return = .4 × (-12) + .6 × (20) = 7.2%

Portfolio AB’s normal return = .4 × (14) + .6 × (12) = 12.8%

Portfolio AB’s boom return = .4 × (25) + .6 × (-10) = 4%

Portfolio AB’s expected return = 0.3 × 7.2+0.5 × 12.8+0.2 × 4 = 9.36%

Portfolio AB’s expected variance = $0.3 \times (7.2-9.36)^2+0.5 \times (12.8-9.36)^2+0.2 \times (4-9.36)^2$

= 1.39968 + 5.9168 + 5.74592

= 13.0624

Portfolio AB’s expected std. dev. = $\sqrt{13.0624}$ = 3.61%

Q4 Listed below are the annual rates of return earned on Stock × and Stock Y over the past 6 years. Which stock was riskier and why?

Year	Stock X	Stock Y
2014	20%	16%
2015	15%	17%
2016	-10%	20%
2017	30%	24%
2018	25%	23%
2019	14%	-10%

ANSWER

Year	Stock X	Stock Y	$(X\text{-Mean})^2$	$(Y\text{-Mean})^2$	
2004	20%	16%	0.001877778	0.0001	
2005	15%	17%	4.44444E-05	0.0004	
2006	-10%	20%	0.065877778	0.0025	
2007	30%	24%	0.020544444	0.0081	
2008	25%	23%	0.008711111	0.0064	
2009	14%	-10%	0.000277778	0.0625	
Average	16%	15%	0.019466667	0.016	Variance
			13.95%	12.65%	Std. dev.

We calculate each stock's average return, variance, and standard deviation over the past 6 years and compare their risk per unit of return i.e. σ/Average

	Stock X	Stock Y
Average return	16%	15%
Standard Deviation	13.95%	12.65%
Standard Deviation / Average Return	0.8718%	0.8433%

Stock × was riskier than Stock Y since it had the higher Standard Deviation of the two, and its average return was not much higher than Stock Y's average return resulting in 0.872% risk per unit of return versus Stock Y's 0.843%risk per unit of return.

Q5 Use the information in the following to answer the questions below.

State of Economy	Probability of State	Return on R in State	Return on S in State	Return on T in State
Boom	.15	0.040	0.280	0.450
Growth	.25	0.040	0.140	0.275
Stagnant	.35	0.040	0.070	0.025
Recession	.25	0.040	-0.035	-0.175

a. What is the expected return of each asset?

ANSWER (a)

Expected Return R = 0.15 × 0.04 + 0.25 × 0.04 + 0.35 × 0.04 + 0.25 × 0.04
= 0.0060 + 0.0100 + 0.0140 + 0.0100= 0.0040 or 4.0%

Expected Return S = 0.15 × 0.28 + 0.25 × 0.14 + 0.35 × 0.07 + 0.25 × -0.035
= 0.0420 + 0.0350 + 0.0245 – 0.0088 = 0.0928 or 9.28%

Expected Return T = 0.15 × 0.45 + 0.25 × 0.275 + 0.35 × 0.025 + 0.25 × -0.175
= 0.0675 + 0.0688 + 0.0088 – 0.0438 = 0.1013 or 10.13%

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b. What are the variances and standard deviations of each asset?

ANSWER (b)

σ^2 (R) = 0.15 × $(0.04 – 0.04)^2$ + 0.25 × $(0.04 – 0.04)^2$ + 0.35 × $(0.04-0.04)^2$ + 0.25 × $(0.04 – 0.04)^2$
= 0.15 × 0.0000 + 0.25 × 0.0000 + 0.35 × 0.0000 + 0.25 × 0.0000
= 0.0000+ 0.0000 + 0.0000 + 0.0000 = 0.0000

Standard Deviation of R = $(0.0000)^{1/2}$ = 0.0000 or 0.00%

σ^2 (S) = 0.15 × $(0.28 – 0.0928)^2$ + 0.25 × $(0.14 – 0.0928)^2$ + 0.35 × $(0.07 – 0.0928)^2$ + 0.25 × $(-0.035 – 0.0928)^2$
= 0.15 × 0.0350 + 0.25 × 0.0022 + 0.35 × 0.0005 + 0.25 × 0.0163
= 0.0053 + 0.0006 + 0.0002 + 0.0041 = 0.0101

Standard Deviation of S = $(0.0101)^{1/2}$ = 0.1004 or 10.04%

σ^2 (T) = 0.15 × $(0.45 – 0.1013)^2$ + 0.25 × $(0.275 – 0.1013)^2$ + 0.35 × $(0.025 – 0.1013)^2$ + 0.25 × $(-0.175 – 0.1013)^2$
= 0.15 × 0.1216 + 0.25 × 0.0302 + 0.35 × 0.0058 + 0.25 × 0.0763

= 0.0182 + 0.0075 + 0.0020 + 0.0191 = 0.0469

Standard Deviation of T = $(0.0469)^{1/2}$ = 0.2166 or 21.66%

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c. What is the expected return of a portfolio with equal investment in all three assets?

ANSWER (c)

Expected Return Portfolio = 0.3333 × 0.04 + 0.3333 × 0.0928 + 0.3333 × 0.1013

= 0.0133 + 0.0309 + 0.0338 = 0.0780 or 7.80%

OR

First determine the portfolio's return in each state of the economy with the allocation of assets at 1/3 in R, 1/3 in S, and 1/3 in T.

Expected Return Portfolio in Boom = 0.3333 × 0.04 + 0.3333 × 0.28 + 0.3333 × 0.45

= 0.0133 + 0.0933 + 0.1500 = 0.2567 or 25.67%

Expected Return Portfolio in Growth = 0.3333 × 0.04 + 0.3333 × 0.14 + 0.3333 × 0.275

= 0.0133 + 0.0467 + 0.0917 = 0.1517 or 15.17%

Expected Return Portfolio in Stagnant = 0.3333 × 0.04 + 0.3333 × 0.07 + 0.3333 × 0.025

= 0.0133 + 0.0233 + 0.0083 = 0.0450 or 4.50%

Expected Return Portfolio in Recession = 0.3333 × 0.04 + 0.3333 × (-0.035) + 0.3333 × (-0.175)

= 0.0133 – 0.0117 – 0.0583 = -0.0567 or -5.67%

Now take the probability of each state times the portfolio outcome in that state:

Expected Return Portfolio = 0.15 × 0.2567 + 0.25 × 0. 1517 + 0.35 × 0.0450 + 0.25 × -0.0567

= 0.0385+ 0.0379 + 0.0158 – 0.0142 = 0.0780 or 7.80%

Note that either way produces the same expected return but that for the variance calculation the portfolio returns in the three economic states are needed.

d. What is the portfolio's variance and standard deviation using the same asset weights in part c?

ANSWER (d)

Variance of Portfolio = $0.15 \times (0.2567 – 0.0780)^2 + 0.25 \times (0.1517 – 0.0780)^2 + 0.35 \times (0.045 – 0.0780)^2 + 0.25 \times (-0.0567 – 0.0780)^2$

= 0.15 × 0.0319 + 0.25 × 0.0054 + 0.35 × 0.00.11 + 0.25 × 0.0181

= 0.0048+ 0.0014 + 0.0004 + 0.0045 = 0.0111

Standard Deviation of Portfolio = $(0.0111)^{1/2}$ = 0.1052 or 10.52%

Q6 B C Inc. originally purchased the rookie card of Aaron for Rs35.00. After holding the card for five years, B C auctioned off the card for Rs180.00. What are the holding period return and the annual return on this investment?

ANSWER

Holding Period Return = (Rs180 – Rs35) / Rs35 = 4.1429 or

414.29% Annual Percentage return= HPR/n =

414.29%/5=82.86%

EAR = $(1 + 4.1429)^{1/5}$ – 1 = 1.3875 – 1 = 0.3875 or 38.75%

OR

Using a financial calculator:
PV= -35; FV = 180; N = 5; PMT = 0; I = 38.75% ==>

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