Some Examples Related to Subring →

\# Prove that the set of Matrices $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a Subring of the ring of 2x2 Matrices with integral domain.

Solution → Let R be the Set of 2x2 Matrices with integral ~~domain~~ elements.

and R is a ring

Let S be the Set of Matrices of type $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$, where a, b, c are integers.

$\therefore$ S is a Subset of R.

As $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in S$ therefore $S \neq \phi$

Let $A = \begin{bmatrix} a_1 & b_1 \\ 0 & c_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 \\ 0 & c_2 \end{bmatrix}$ be any two Elements of M

$$A - B = \begin{bmatrix} a_1 - a_2 & b_1 - b_2 \\ 0 & c_1 - c_2 \end{bmatrix}$$

and $$A \cdot B = \begin{bmatrix} a_1 & b_1 \\ 0 & c_1 \end{bmatrix} \begin{bmatrix} a_2 & b_2 \\ 0 & c_2 \end{bmatrix} = \begin{bmatrix} a_1 a_2 & a_1 b_2 + b_1 c_2 \\ 0 & c_1 c_2 \end{bmatrix}$$

Since the Elements of A−B and AB are integers

So $A - B \in S$ and $A \cdot B \in S$

thus we have $A, B \in S \implies A - B \in S$ and $A \cdot B \in S$

Hence S is a Subring of R.

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Q→ If R is the ring of integers and S is a Subset of R such that

$$S = \{ \ldots, -3K, -2K, -K, 0, K, 2K, 3K, \ldots \}$$

where K is a fixed integer, then Show that S is a Subring of R.

Solution → As $0 \in S$ therefore $S \neq \phi$

Let $x = aK$ and $y = bK$ where $a, b \in \mathbb{Z}$

$\therefore$ $x - y = (a-b)K \in S$ as $a-b \in \mathbb{Z}$

and $x \cdot y = abK^2 = (abK)K \in S$ as $abK \in \mathbb{Z}$

thus If $x, y \in S \Rightarrow x - y \in S$ and $x \cdot y \in S$

Hence S is a Subring of R.

Q→ Prove that the Set of integers is a Subring of ring of rational Numbers.

Solution→ Let $\mathbb{Z}$ be the Set of integers and Q be the ring of rationals.

$\therefore$ $\mathbb{Z}$ is a Subset of Q.

$$0 \in \mathbb{Z} \Rightarrow \mathbb{Z} \neq \phi$$

Let a, b be any two Element of $\mathbb{Z}$

$\therefore$ $a - b \in \mathbb{Z}$ and $ab \in \mathbb{Z}$

Hence $\mathbb{Z}$ is a Subring of Q.

ideals →

(1) **Right ideal** → A non-Empty Subset S of a ring R is called a right ideal of R if (i) S is a Subgroup of R with respect to addition

(ii) $sr \in S$ for all $s \in S$ and $r \in R$

(2) **left ideal** → A non-Empty Subset S of a ring R is called a left ideal of R if (i) S is a Subgroup of R with respect to addition.

(ii) $rs \in S$ for all $s \in S$ and $r \in R$

(3) **Ideal** → A non-Empty Subset S of a ring R is called an ideal of R if

(i) S is a Subgroup of R with respect to addition.

(ii) $s \in S$ and $r \in R \Rightarrow sr \in S$ and $rs \in S$

Or

A non-Empty Subset S of a ring R is called an ideal of R if

(i) $a, b \in S \Rightarrow a - b \in S$

(ii) $a \in S$ and $r \in R \Rightarrow ar \in S$ and $ra \in S$

Note :-

1.) S is called a two sided or both sided ideal of R if it is both right and left ideal. infact, if we say S is an ideal of R, it would mean, S is both sided ideal of R.

2.) In a commutative ring, every right ideal or left ideal is two sided.

3.) An ideal S of a ring R is a subring of R. But every subring of R is not an ideal. This is because of the stronger closure property for an ideal than the subring.

for a subring, the condition is that for all $r, s \in S \Rightarrow rs \in S$ whereas ideal requires a stronger condition i.e for all $r \in R$, for all $s \in S \Rightarrow$

$rs \in S$ and $sr \in S$

4.) In every ring R, the ideal $\{0\}$ and R are called trivial or improper ideals. Any ideal except these two is called a non-trivial ideal or a proper ideal.