Indira Gandhi
National Open University
School of Education

BES-143
Pedagogy of Mathematics

Block

1

UNDERSTANDING THE DISCIPLINE OF MATHEMATICS

EXPERT COMMITTEE

SPECIAL INVITEES (FACULTY OF SOE)

Course Coordinators : Prof. M.C. Sharma, SOE, IGNOU
Dr. Anjuli Suhane, SOE, IGNOU

COURSE TEAM

Course Contribution

Unit (1 and 2)
Mr. Ajith Kumar C
Assistant Professor, SOE, IGNOU, New Delhi

Unit (3 and 4)
Dr. Vijayan K.
Assistant Professor
Department of Teacher Education, NCERT, New Delhi

Content Editing
Prof. C.P.S. Chouhan
Former Dean
Faculty of Education, AMU, Aligarh

Language Editing
Prof. Amitav Mishra
SOE, IGNOU, New Delhi

Format Editing
Dr. Anjuli Suhane
Assistant Professor
SOE, IGNOU, New Delhi

Proof Reading
Dr. Anjuli Suhane
Assistant Professor
SOE, IGNOU, New Delhi

PRODUCTION

Prof. Saroj Pandey
Director
SOE, IGNOU, New Delhi

Mr. S.S. Venkatachalam
Assistant Registrar (Publication)
SOE, IGNOU, New Delhi

April, 2017

ISBN-

Further information on the Indira Gandhi National Open University courses may be obtained from the University's Office at Maidan Garhi, New Delhi-110068.

Printed and published on behalf of the Indira Gandhi National Open University, New Delhi by Director, School of Education, IGNOU, New Delhi.

Laser Typeset by Tessa Media & Computers, C-206, A.F.E.-II, Okhla, New Delhi.

Printed at:

Course : BES-143 Pedagogy of Mathematics

COURSE INTRODUCTION

There is a common notion among student community that 'Mathematics is always a difficult subject' and they approach Mathematics with a lot of fear. The fear towards Mathematics learning may be due to various reasons. First of all, Mathematics is taught in a mechanical style to a large extent. Secondly, the teachers are not making use of effective pedagogical skills in delivering mathematics content to the learner. Thirdly, learning situations/lesson plans/ activities are not well designed and are not prepared keeping in view the individual differences of the learner. If you want to become an effective teacher, these factors must be taken into consideration and you must equip yourself with adequate knowledge both in content and pedagogical aspects related to Mathematics teaching-learning. Keeping these factors in mind, we have designed this particular course giving due consideration to basic aspects such as 'what is Mathematics', 'why to teach Mathematics' and 'how to teach Mathematics'.

Objectives of the Course: After completion of the course, student-teacher should be able to:

- develop a critical understanding of changing perspectives of Mathematics,
- appreciate the Indian contribution in development of Mathematics,
- understand the nature of Mathematics and its place in curriculum,
- construct teaching-learning objectives for Mathematics,
- identify and use appropriate approach for teaching-learning of Mathematics,
- select and integrate suitable learning resources to facilitate learning in Mathematics,
- construct and use appropriate assessment tools for assessing learners' progress in Mathematics,
- appreciate the role of innovations and research in expanding knowledge domain of Mathematics, and
- adopt appropriate strategies for professional development of the self.

This course contains four blocks and you will come across 4 or 5 Units in each block. The following are the blocks included in this particular course of study:

Block 1: Understanding the Discipline of Mathematics

Block 2: Teaching-Learning of Mathematics

Block 3: Content Based Methodology-I

Block 4: Content Based Methodology-II

The first block of this course deals with nature of Mathematics discipline, aims and objectives of teaching–learning mathematics, ways and means to develop mathematical thinking of learners, various ways of learning Mathematics, processes involved in Mathematics learning and Mathematics in school curriculum.

The second block deals with the teaching learning process of Mathematics. Pedagogical shift; approaches for teaching learning Mathematics: how to design learning experiences, desirable characteristics of a good learning experience in Mathematics; how to involve learner in teaching learning process; preparation of Unit and lesson plan; learning through activities: Mathematics quiz, riddle and tricks; learning resources and ICT for Mathematics teaching-learning; role and various tools and techniques of assessment in Mathematics; preparation of achievement test; teaching-learning and assessment strategies for differently-abled learner and professional development programmes for Mathematics teachers are discussed and illustrated at length in this block.

Third and fourth blocks focus on content /concepts. In both these blocks, teaching-learning process and different modes of evaluation of these concepts have been dealt. Third block deals with various aspects of Algebra and fourth block deals with the fundamental aspect of Statistics, Probability, Geometry, Trigonometry, Mensuration and Coordinate Geometry.

Once you complete all the blocks of this course, we hope you would be able to consolidate your understanding of various mathematical concepts at secondary school level and design and organize various learning activities to teach Mathematics to secondary school level learners.

BLOCK 1 UNDERSTANDING THE DISCIPLINE OF MATHEMATICS

Block Introduction

For being a successful teacher, who can motivate and facilitate learner's learning, it is important to acquire pedagogical knowledge as well as mathematical knowledge. Teachers need to understand the blending of content and pedagogy that enables them to organize and represent content in such a way to address the diverse ways in which learners respond to instruction. It is important that teachers are able to relate to the nature of mathematical knowledge. Every discipline has aspects which enables us to understand the content better. In this Block we will be addressing these aspects. The nature of Mathematics affects the way we learn Mathematics, the way teacher teach it, and will affect the way the learner view Mathematics! As a teacher, moving from one topic to the next, from one skill to the next, very rarely we stand back to take a broader view of Mathematics and think 'What is Mathematics?' There are

different things that can be learnt from school Mathematics. Of course one learns mathematical knowledge, but one also learns mathematical thinking and the way Mathematics is applied in our day-to-day life. This block attempts to make you realize the importance of these aspects. This particular block contains four Units, namely:

Unit 1: Nature and Scope of Mathematics

Unit 2: Aims and Objectives of Teaching –Learning Mathematics

Unit 3: How Learner Learn Mathematics

Unit 4: Mathematics in School Curriculum

The first Unit, **Nature and Scope of Mathematics** acquaint you with some of the key aspects of discipline of Mathematics like concept and nature of Mathematics, validation process of mathematical statements: proof, counter-example, conjecture, invalidity of arguments, simple fallacies and Creative thinking in Mathematics. This Unit attempts to make you familiar with the process of doing Mathematics and how to enable learner to carry out the process efficiently and creatively.

Why do we teach Mathematics? What are its aims and objectives? The second Unit, '**Aims and Objectives of Teaching –Learning Mathematics'** attempt to answer some of the relevant questions associated with Mathematics teaching-learning. The primary objective of Mathematics learning is to equip learner with the power of mathematisation. Methods to develop the power of mathematisation have been discussed in this Unit. This Unit will give you an idea on the strategies to improve learners' reasoning skill and methods to approach mathematical problems. This Unit also explains the necessity to teach Mathematics by correlating with other subjects.

Learning Mathematics is a big challenge for primary learner. As a teacher you should develop knowledge on the methods by which learner learn Mathematics. So the first section of the third Unit, **'How Learner Learn Mathematics'** explores the different methods and ways learner use in order to understand mathematical concepts. Piaget's, Bruner's, and Vygotsky's views on learning of mathematical concepts are discussed in the second section of this Unit. Problem Solving, Patterning, reasoning, Abstraction, Generalization, Argumentation, Justification as the processes involved in learning of Mathematics are explained in the last section of this Unit.

The fourth Unit, **Mathematics in School Curriculum** acquaints you with some of the key social, mathematical and practical aspects of Mathematics in school curriculum. Core areas of concern in school Mathematics, curricular choices at different stages of school Mathematics education and principles of formulating Mathematics curriculum are discussed in this Unit. Subject centered to behaviourist to constructivist approach of curriculum development are also explained in detail in this Unit.

UNIT 1 NATURE AND SCOPE OF MATHEMATICS

Structure

1.1 INTRODUCTION

Mathematics is a subject that finds application in every walk of our life. Knowingly or unknowingly, people use concepts of Mathematics in their daily life. Considering the relevance of Mathematics, it is treated as one of the basic and compulsory subjects in school curriculum. In the elementary school curriculum, learning basic concepts of Mathematics is emphasised. As and when children reach higher classes, the complexity of mathematical concepts gets widened. It is commonly observed that many children have a belief that Mathematics is, in a way, difficult to learn and understand. In this context, teachers have a great role to play. A successful teacher of Mathematics should have profound knowledge of nature and theoretical concepts in Mathematics so as to help children in effective learning.

The nature of Mathematics includes mathematical ideas progress from concrete to abstract; grow from particular to general and its knowledge is conceptual as well as procedural. Similarly, in Mathematics we come across 'definitions' that describe concepts; 'examples' to illustrate procedures; 'theorems' to state valid results; 'conjecture' that talks about mathematical statements for which proofs are to be worked out but which seem plausible, and 'counter example' to disprove statements. A teacher who has adequate knowledge on these aspects of Mathematics would be able to organise learning activities in a more effective way. This unit will elaborate on these concepts in an extensive manner.

1.2 OBJECTIVES

After completing this unit, you will be able to:

- describe Mathematics as a discipline;
- explain the nature of Mathematics;
- explain the key processes in mathematical reasoning;
- describe about a mathematical proof;
- describe about a conjecture in Mathematics;
- describe a counter example in Mathematics;
- enable children to develop and defend mathematical arguments efficiently; and
- identify ways which make children creative.

1.3 DISCIPLINE OF MATHEMATICS

The development of the discipline of Mathematics dates back in thousands years. Starting from the prehistoric period to the twentyfirst century, much development has taken place in the field of Mathematics. The Babylonian, Egyptian, Greek, Chineese and Indian mathematicians have contributed remarkably to the field of mathematics. To name a few, Pythagoras, Fibonaaci, Aryabhata, Euclid, Archimedes and Rene Descartes have contributed significant inputs to the field of Mathematics in ancient times. During the sixth century B.C. **Pythagoreans (Pythagoras and his followers) coined the term Mathematics. Mathematics is derived from the Greek word 'Ganita' which means 'inclined to learn'.**

'Mathematics' is a broad term that encompasses many branches and components. It is this view that makes the word 'Mathematics' hard to define. Even then, we would say, **Mathematics is a science that involves dealing with numbers, different kinds of calculations, measurement of shapes and structures, organisation and interpretation of data and establishing relationship among variables, etc.** Despite of such broadness, a few definitions of Mathematics are given below:

- "Mathematics should be visualised as the vehicle to train a child to think, reason, analyse, and articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and meaning" (National Policy on Education, 1986).
- "We cannot overstress the importance of Mathematics in relation to science, education and research. This has always been so, but at no time has the significance of Mathematics been greater than what it is today-it is important that deliberate efforts are made to place India on the world map of Mathematics within the next two decades or so" (National Education Commission or Kothari Commission, 1964-66).
- "Mathematics, as an expression of the human mind, reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality" (Courant and Robbins, 1996)

The definitions given above will enable one to conclude that; Mathematics is a study of patterns, numbers, geometrical objects, data and chance. It is a diverse discipline that deals with data analysis, integration of various fields of knowledge, involves proofs, deductive and inductive reasoning and generalisations, gives explanation of natural phenomena and human behaviour. Mathematics also helps to understand the world around us by exploring the hidden patterns in a systematic and organised manner; and it has universal applicability.

Mathematics is considered as a formally organised subject of study, and hence, it is treated as a discipline. In Mathematics, we deal with various branches such as arithmetic, algebra, geometry, trigonometry, etc. The branches of Mathematics find application in our daily life. Keeping these views, it is said that Mathematics has gained relevance and popularity as a useful subject. Mathematics is classified broadly into two types, which are given below:

Pure Mathematics: A study of the basic concepts and structures for the purpose of a deeper understanding of the subject. Pure Mathematics deals with the basic information/facts of Mathematics where various concepts, proofs and theories, etc. are discussed. For example, the theoretical knowledge concerning arithmetical operations such as addition, subtraction, multiplication and division are part of it.

Applied Mathematics: Applied mathematics is an abstract science of numbers, quantity and space as applied to other disciplines such as Physics and Engineering. The Pure Mathematics when utilized to solve different problems either mathematical or life is termed as applied Mathematics. For example, children study the concept of 'addition' which is explored while buying food items from a grocery shop' the concept of 'interest' is used to calculate the interest on money deposited in banks, etc.

1.3.1 Concept and Nature of Mathematics

People started using the concepts of Mathematics long back, and it is the backbone of modern civilisation. Nowadays, Mathematics is used by every individual. Consider an example of a boy who buys groceries from a shop. The boy may ask, "Uncle! Can you please give me 1 kg sugar?" Then the salesman would use his weighing balance to weigh the sugar. For that, he will put 1 kg weight in one pan of the balance, and in the other, he will put 1 kg sugar. He will put sugar until it gets balanced with the 1 kg weight. Then, he will hand over the sugar to the boy, and in return, the boy pays the price for 1 kg sugar. What did you understand from this example? Did you find any Mathematics here?

Definitely! in the given example, Mathematics is involved at many places. For instance, the boy asks 1 kg sugar. Mathematics is involved in it. The quantity '1kg' sugar is a kind of measurement. When the salesman weighs, he makes use of the principle of balance (in a way, balancing equations, equal to symbol etc.). Similarly, the payment of price of sugar also involves Mathematics. Thus, Mathematics is associated in our day-to-day activities. This helps us to understand its nature.

What is the nature of Mathematics? First, **Mathematics is the "queen of all sciences" and its presence is there in all the subjects**. Mathematics acts as the basis and structure of other subjects. These views have brought in relevance of Mathematics to be considered as one of the core subjects of school curriculum. Second, **Mathematics is more than computation**. What does it mean? For example, if someone is asked, "How many apples can be filled in a square shaped box"? He/she may say, "approximately 100". This answer is an approximation. But, for anyone to get the exact answer, he/she should be well versed with the mathematical concepts such as, the shape of box, the style of packing etc. Thus, we would conclude that Mathematics is more than mere computations. Mathematics gives us clear and correct answers through calculations.

Let us now discus, the three other aspects that are crucial in understanding the nature of mathematical ideas. Mathematical ideas grows from concrete to abstract, particular to general and its knowledge is conceptual as well procedural.

Concrete to Abstract: How do students develop the concept of 'roundness' or 'circle' or 'sphere'? People see various objects around them which may have different shapes. But as students see objects like ball, orange, water melon, etc. they relate such objects and gradually arrive at a conclusion that, a few objects are similar in shape. Thus the concrete experiences help students to develop the concept of 'roundness'. As the frequency of dealing with concrete experiences increases, gradually, concepts like 'circle', 'sphere' etc. comes to their mind. At this stage the abstractness of 'concept' expands since roundness is different from, circle which in turn is different from sphere. At a later stage, the abstractness grows to various other related concepts of circle and sphere such as diameter, radius, perimeter, volume, and so on. Thus we may conclude that, every mathematical concept gives rise to more such concepts and hence abstractness widens.

Particular to General: If somebody says closed figure, what will come to our mind? This is individual dependent; to one he/she may feel a circle, for other it may be square and so on. Suppose an individual see different closed figures having only four sides, it may help him/her to conclude that, all those figures have four sides and are closed. Seeing similar figures on various occasions (particular cases), would help him/her to generalise and develop a new concept i.e. those figures have four sides and are closed ; such figures are called quadrilaterals. Thus experiences with particular cases will broaden the abstractness and help individuals to arrive at a general concept.

Conceptual and Procedural Knowledge: A concept may be generalised idea. The knowledge of an individual on different concepts of Mathematics is called conceptual knowledge. For example, the concept of numbers, arithmetic operations, shapes, fractions, data, chance, etc. or we can also say that conceptual knowledge is nothing but understanding of the concepts like definitions, rules etc. When students understand a concept in meaningful way, they are more likely to be able to correctly apply it in various situations. The knowledge of procedures followed in solving mathematical problems is called procedural knowledge. For example, the procedure followed in drawing perpendicular bisector of a given line segment.

Let us now discuss how these two mutually dependent aspects help in learning. Let us consider the problem of finding area of the circle. To find the area, children should have knowledge (conceptual knowledge) of the concepts like area, pi (π), diameter of the circle, radius of the circle, multiplication and the formula for finding area of the circle. Those children, who have conceptual knowledge and know the procedures to calculate area of the circle, may be able to calculate (procedural knowledge) the area of the circle by substituting appropriate values in the formula, $A = \pi r^2$

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

1) "Mathematics is involved in day-to-day activities". Justify the statement with an example.

....

....

....

....

....

2) How does Mathematics find application in other subjects?

....

....

....

....

....

1.4 HOW KNOWLEDGE IN MATHEMATICS IS VALIDATED AND PROVED?

Learning Mathematics is concerned with both comprehending the facts, theories, rules and laws, and with the aim of learning mathematics which is much broader. As a teacher, one should make his/her students understand that mathematics has various applications in daily life. Similarly, it is the duty of the teacher to convince students that mathematics is not a subject to be learned in by rote-learning. Children do come across situations to solve problems, and for the same, the mathematical reasoning is used. To arrive at solution, either the inductive or deductive reasoning is used. While reasoning, children organise data, evaluate situations, formulate relations, solve problems and identify situations that have practical applications. Knowingly or unknowingly, children make use of such situations and, through such stages in solving problems. Let us discuss an example:

Example

Suppose a teacher wants her/his students to draw the bisector of a given angle, what will she/he does? She/he poses it as a problem before the entire class. Then, the students will be engaged in the learning activity. They may follow the steps given below:

Step 1: First, two rays namely, AB and AC are drawn.

Step 2: Taking A as centre, two arcs of any but equal radius, are drawn that intersect the rays AB and AC at D and E respectively.

Step 3: Taking D and E as centres, two arcs with radius more than ½ DE are drawn that intersect each other at a point F.

Step 4: Draw the ray AF. This is the required bisector of the angle BAC.

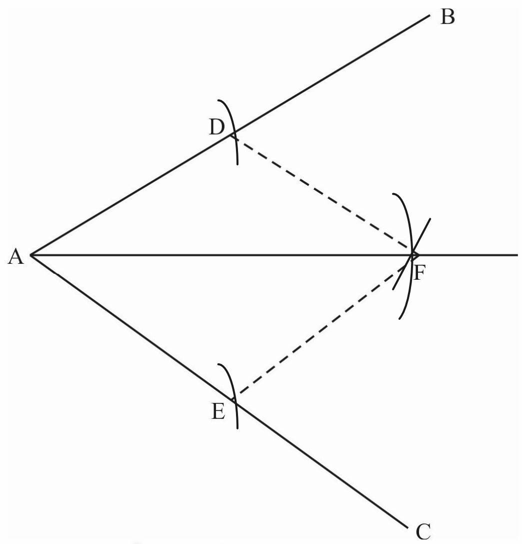


Fig. 1.1

How does the ray become the required bisector of an angle? This can be explained as follows:

First, draw the lines DF and EF and consider the triangles ADF and AEF.

Then we have, AD = AE (Radii of the same arc)

And DF = EF (Arcs with equal radii)

And AF = AF (Common side)

Then Δ ADF $\cong$ Δ AEF (SSS rule)

Therefore $\angle$DAF = $\angle$EAF (CPCT)

What conclusions we can draw from this activity? Here, the children are faced with problem of drawing the bisector of a given angle. In order to find the solution, the children have used their previous knowledge, a few mental calculations, and reasoning while drawing different arcs and rays, created a few relations, equated a few formulas and finally succeeded in finding the solution to the problem i.e. drawing the bisector of an angle. As teachers, we should be aware of the fact that students make use of various processes to arrive at solutions to the problems that they encounter, and such process are considered to be the key processes in mathematical reasoning. **Technically, we name these processes as generalisations, conjectures, counter examples, hypotheses and proofs.** These are the key processes used to build and validate mathematical knowledge.

Imagine that a 14 years boy is walking across a street. Every possibility is there for him to see different kinds of shops. It may be a grocery shop, vegetable shop, meat shop and so on. If the boy walks about 1km and sees that most of the shops are selling grocery items, then, there is every possibility to conclude that the market sells only grocery items. In a way, it is a kind of **generalisation**. In Mathematics also, we use to make generalisation which is one of the ways of reasoning. **By generalisation, we mean to deduce an observation about something, which may be true or false**. There is equal possibility that a generalisation may go wrong or right.

Before, discussing the other key processes in detail, let us briefly touch upon each one of them. In the above example, if the child sees a shop that sells both groceries and vegetables, then he may **conjecture** that, the market is not grocery market. Similar to generalisation, the conjecture may be right or wrong. Conjecture also needs to be checked for its truthfulness. Suppose the shops have grocery items at the front side of the shop and rest of the spaces contain vegetables, fruits and a coffee bar, then there is a possibility of concluding that the market is grocery market. The conclusion in this case is called **counter example**, which is false.

There are situations whereby the child may confuse in judging shops that sell grocery items and sweet (bakery) items. In such cases, the child fails to generalise whether the market is grocery market or bakery market or the combination of both. In mathematical reasoning, such a situation is treated as a **special case**. Sometimes, the theories and generalisations that we arrive at may not fit into special cases. As discussed in the above paragraph, at times we get confuse in judging about the market is grocery market or something else, specially, the shops that sell both groceries and vegetables. Even then, we generalise that, the market is grocery market. In Mathematics, this is called **hypothesis**. We hypothesise that the market is grocery market. The hypothesis remains hypothesis until we provide a convincing argument or explanation, which we call the **proof** in Mathematics to justify or reject it.

Now, let us discuss these processes in detail.

1.4.1 Conjecture

At times, children explore the relationship among various components, identify patterns and develop some statements (or arrive at results) based on the observations. But, there is no guarantee that those statements (results) are right or wrong. In order to accept it as an organised body of knowledge, we need to prove it through arguments. It is at this stage that children generally form series of logical arguments and explanations to prove or disprove the result developed. The same is observed in solving mathematical problems or investigating relationships among components to arrive at results. While children try to find solutions to mathematical problems, they tend to build up conjectures, which for them seem to be true until concrete evidence is produced.

"The word conjecture is often used in the context of mathematical problem-solving and investigating". It refers to an assertion that something might be true, at the stage when there has not yet been produced the evidence necessary to decide whether or not it is true. A conjecture is therefore usually followed by some appropriate mathematical process of checking. This experience of conjecturing and checking is fundamental to reasoning mathematically" (Haylock, 2010).

Let us discuss a classroom situation where the children attempt to develop a conjecture that goes right at a later stage. Remember, conjecture may be proved true or false at a later stage. Examine the classroom interaction given below:

Teacher : Hi children! Today, we are going to study something about numbers.

Students : Yes, Madam. (Together with loud noise)

Teacher : I am going to write a few numbers on the blackboard. Your task is to identify the rational numbers among them. Is it clear to you?

Students : Yes Madam!

Then the teacher writes the following numbers on the black board;

12, 1/3, 6/8, 54/34, 98, 23, 5/8, 34/98, 1, 60

Teacher : Do you see the numbers? Now, let me ask, Arathy! Can you identify the rational number among them?

Arathy : Madam, the rational numbers are 1/3, 6/8, 54/34, 5/8 and 34/98.

Teacher : Are you sure?

Aarathy : Yes madam!

The discussion continues. What did you observe in this example? Here, the student has made a conjecture that among the given numbers, the ones which are in p/q form are rational numbers. The student had the feeling that her answer is right. These kinds of judgments/statements are called conjectures. But, this conjecture is wrong. To correct the children teacher explained as follows:

Teacher : A number is said to be rational number when it is written in the form p/q where p and q are integers, and q is not equal to zero. Not only 1/3, 6/8, 54/34, 5/8 and 34/98 are rational numbers, but also 12, 98, 23, 1, 60 are rational numbers. Why? It is because, we can write, 12, 98, 23, 1, 60 as 12/1, 98/1, 23/1, 1/1 and 60/1 respectively. Thus, they are in the form of p/q and hence, are also rational numbers.

The children get convinced that their assumption was wrong and realised that all the whole numbers written on the board were rational numbers. As teacher, you should organise situations to get conjectures developed by students and to test them at the same time. This helps children improve their level of thinking and skill in formulating conjectures.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

3) What is a conjecture? Give an example.

....

....

....

4) List the processes used to validate mathematical knowledge?

....

....

....

....

1.4.2 Generalisation

Being a teacher, have you encountered situations, wherein students make some kind of generalisation? Knowingly or unknowingly, children formulate generalised statements, especially, in Mathematics classrooms. May be, in the first instance, we can consider such statements as 'conjecture'. The conjectures, at a later stage, are proved or disproved. There are lots of situations that provide opportunity for children to form generalisations. The teacher should organise such situations so that children may develop their own statements and try to judge such thoughts.

Let us see how, Ms. Manasi, a Mathematics teacher of ninth standard made her students develop generalisation. She was teaching the concept of 'circles'. The intention of the teacher was to make children form statements/generalisations related to the properties of a circle. To achieve the aim, she distributes the following figures amongst children. Then, children were asked to describe the pictures. Before starting the activity, Ms. Manasi grouped her children into ten groups.

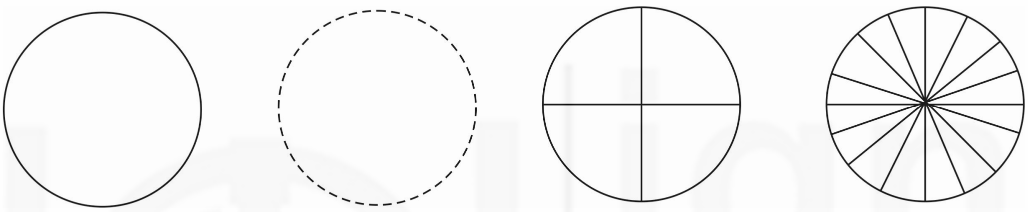


Fig.1.2

After assigning the group task, children were engaged in completing the task. In order to build a competitive spirit, the teacher announced that there would be prizes for those groups that complete the task within 10 minutes.

After 10 minutes, the teacher asked children to present their observations. Let us analyse observations of a few groups. Among the 10 groups, the observations of the groups 3 and 8 are given below:

Point No	Observation- Group 3	Observation- Group 8
1	All are circles.	Circle is made up of points.
2	There are solid circles and empty circles.	The distance from centre of the circle to the boundary is equal at all points.
3	A circle always has a central point.	The circle can be divided into two half circles.
4	In a circle, we can draw a line that touches two points on the circle.	Circle has definite shape.
5	In solid circle, there will be two faces.	Circle can have many diameters.

What do you understand from this activity? Here, Ms. Manasi, made her students to engage in an activity and thereby help them understand the properties of circles. One thing to remember is that, here children have observed the figures. Then, they tried to relate their observations to their previous knowledge about circles. They built an understanding about the pictures, tried to compare circles with other shapes, and generated a few statements. Finally, they arrived at a few general statements, which they thought right. These general statements are referred to as generalisations in mathematical reasoning. This is how mathematical generalisations are developed.

As a concluding remark, teacher explained, “Today we studied about the different properties of circles. I would restate the properties of circles:”

1) A circle is defined as the collection of all points in a plane that are at a fixed distance from a fixed point in the same plane.
2) The fixed point is called the centre of the circle.
3) In every circle, the ‘fixed distance’ is called its radius.
4) The distance from centre to all points on the boundary of circle is equal.
5) A circle also has a diameter.

The teacher highlighted a few properties of the circle. In mathematical terms, we call it as a generalised statement or generalisation. Usually, children’s thinking moves from specific cases to general rule, while they develop generalisation and it is a key process in mathematical reasoning. It is not certain that generalisation would always be true. Thus, generalisation is checked for its validity and conclusions are confirmed. “Generalisation in Mathematics is an assertion that something is true for all the members of the set of numbers or shapes or people. A generalisation may be true (for example, ‘all multiples of 12 are multiples of 3’) or false (for example, ‘no women can read maps’)” (Haylock, 2010). You may note that, ‘always, every, each, any, all, if …then, so on are a few commonly used language notations in framing generalisation.

1.4.3 Counter Example and Special Cases

We have discussed the theoretical considerations underlying conjectures and generalisations. Now, we will discuss ‘counter examples’, another key process in mathematical reasoning.

Suppose, a teacher makes a statement in his/her class that ‘all prime numbers are odd numbers’. Do you think the statement is correct? Definitely not! Because, even though two (2) is an even number, it is prime. Thus, we can’t make a statement like “all prime numbers are odd’. In this example, the number 2 is considered as a special case and a counter example. Thus, the generalisation can be modified as ‘all prime numbers are odd except two’. This description explains the theoretical concept of ‘counter example’ and special cases. In a way counter example is a special case.

“A counter example is a specific case that demonstrates that generalisation is not valid” (Haylock, 2010). In the preceding sections, we have discussed the concepts of conjecture and generalisation. Both, conjecture and generalisation are wild guesses which are validated through logical arguments and proofs. Counter example is used to disprove the validity of generalisation and special cases are employed for it. When we realize that the generalisation developed is wrong; it helps in modifying and reframing so that the knowledge is further validated.

In the following paragraph, we are going to observe a classroom interaction. In this classroom dialogue, the teacher, Mr. Paul, is helping his children develop the concept of counter example through a small example.

Classroom Interaction

Mr. Paul : Hello children! How are you? Have all of you had your breakfast?

Students : Yes sir!

Mr. Paul : What did we study yesterday?

Students : We studied about symmetry of shapes. Sir!

Mr. Paul : That's pretty good. Shall I ask you a question?

Students : Yes sir!

Mr. Paul : I would say, all rectangles have only two lines of symmetry. Isn't it?

On hearing the question a few children got confused, but majority of them felt confident to give the answer. Let us see the responses of the children.

Shyam : Sir, what you said is correct.

Kishore : Sir, rectangle has only two symmetries

Vyshnavi : Yes sir, It is two.

Fayas : Yes sir, two lines of symmetry.

Elina : I too think sir. Rectangle has two lines of symmetries.

Many of the students said that rectangles have two lines of symmetries. But, the teacher didn't stop. He asked children to prove it. While attempting to prove, a student, named Jatin, stood up and said;

Jatin : Sir! The answer is wrong.

Mr. Paul : Oho. Are you sure?

Jatin : Yes sir!

Mr. Paul : That's good. Then how is it? Can you explain?

Jatin : Sir! Square is also a rectangle and it contains four lines of symmetry. Hence, the statement that "all rectangles have only two lines of symmetry" is wrong.

Mr. Paul : Fine. Very good. Then, can you modify the statement.

Jatin : Sir. All rectangles, except squares, have two lines of symmetry.

Mr. Paul : Very good. Keep it up.

The above interaction shows that there are situations in which we may make mistakes in forming generalisation, and it is rectified at a later stage. Counter example is one of the means that help children to disprove statements. In the above example, 'square' is the special case that acted as a counter example to disprove the statement that 'all rectangles have two lines of symmetry'. At a later stage, it is modified as "all rectangles, except squares, have two lines of symmetry". In the classroom, teacher may give such statements so that students will get opportunity to verify it.

1.4.4 Hypothesis and Proof

"The word hypothesis is usually used to refer to a generalisation that is still a conjecture and which still has to be either proved to be true, or shown to be false by means of a counter example. Often a hypothesis will emerge by a process of inductive reasoning , by looking at a number of specific instances that are seen to have something in common, and then speculating that this will always be the case" (Haylock, 2010). Hypothesis is a key process in mathematical reasoning as it acts as the base of knowledge development. It is from hypothesis that we develop new knowledge. How are hypotheses evolved? We may understand the answer through an example given below:

Example

Suppose that a few students are discussing the mathematical concept 'congruent figures'. Let us examine to their classroom discussion:

Student A : I think, we can see congruent figures/materials in our daily life also. Am I right?

Student B : No no,…we can see congruence in triangles only.

Student C : Yes, I agree that congruence is seen only in triangles.

Student D : No.No..I agree to what A said. Congruence is there in many materials.

Student B : Could you prove it?

Student D : Yes, of course. For example, take the case of refills. The refill of any particular company always produces similar refills for a special design of pen. Do you agree? We can check it now. Please take your pens. (These pens were of the same model and company). Now, can you see that all the refills are the same? That means refills are congruent.

The discussion continued and the four of them convinced that there are congruent materials/figures in daily life too. How did they arrive at such a conclusion? The statement "There are congruent figures/materials in our daily life" was simply a guess. In order to prove the statement, they collected the refill and checked for its trueness. In the same way, they checked with other materials like tiffin box, eraser, shoes and so on. Finally, the statement was proved to be correct. This is how mathematical conclusions are arrived at and new knowledge is created.

A hypothesis remains as hypothesis or conjecture until and unless a valid proof is produced. Proof is the process that helps children to develop logical arguments and convincing facts to establish the validity of hypothesis, emerged as a result of inductive reasoning. Thus, in proving a mathematical hypothesis, we move from general case to particular instances, and hence, it is deductive reasoning. "Proof is a mathematical way of reasoning. If generalisation is written in the form of a statement using the 'if …then…' language, a mathematical proof is a series of logical deductions that starts from 'if…' bit and leads to the 'then …' bit. Proof, therefore, involves deductive reasoning" (Haylock, 2010).

Now let us look at, how Mrs. Sharmila made her students form hypothesis and its proof. She asked her children, "What is the sum of angles of a quadrilateral"? To this question, students responded in many ways namely, 180^0, 270^0, 360^0, and so on. These responses are hypotheses. Then, teacher asked each student to prove it. Those students who answered 180^0, 270^0 found that their answers were wrong. While the students who answered 360^0 found their answer correct. Her solution goes like this:

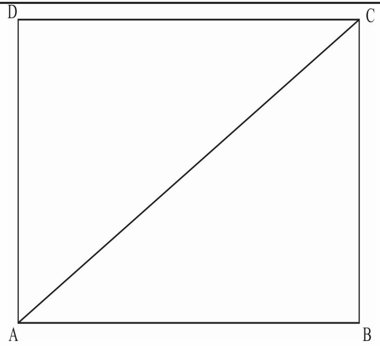


Fig. 1.3

Let ABCD be a quadrilateral and AC be its diagonal.

Then, what is the sum of angles in Δ ADC?

We know that $\angle$ DAC + $\angle$ ACD + $\angle$ D = 180°……………… (1)

Similarly, in ΔABC, $\angle$ CAB + $\angle$ ACB + $\angle$ B = 180°……… (2)

Adding (1) and (2), we get

$\angle$DAC + $\angle$ ACD + $\angle$ D + $\angle$ CAB + $\angle$ ACB + $\angle$ B = 180° + 180° = 360°

Also, $\angle$ DAC + $\angle$ CAB = $\angle$ A and $\angle$ ACD + $\angle$ ACB = $\angle$ C

So, $\angle$ A + $\angle$ D + $\angle$ B + $\angle$ C = 360°.

i.e., the sum of the angles of a quadrilateral is 360°

In this example, a particular student stated the hypothesis, performed logical steps and succeeded in proving the statement 'the sum of the angles of a quadrilateral is 360°'. This is called the 'angle sum property of a quadrilateral'. So, you may also give problems, for which students may develop hypotheses and make an effort to prove it.

1.4.5 Fallacies

Fallacy is an incorrect or misleading notion or opinion based on inaccurate facts or invalid reasoning or we can say that **fallacy** is some apparently logical transition that people think can be done, but that really cannot be done. It is like an error in thinking. As a teacher, you have to identify what type of general fallacies your students have. Then, you can find ways that help children avoid these fallacies/mistakes. One such fact is that we cannot divide a number by zero. Very often we forget to apply this fact which leads to fallacy. Let us look at the following solution:

$a = b.$

$a^2 = ab,$ by multiplying by a.

$a^2 + a^2 = a^2 + ab,$ by adding a^2.

$2a^2 = a^2 + ab,$ simplifying LHS.

$2a^2 - 2ab = a^2 + ab – 2ab,$ by subtracting 2ab.

$2a^2 - 2ab = a^2 - ab,$ simplifying RHS.

$2(a^2 – ab) = a^2 - ab,$ taking 2 common in LHS.

$2 = 1$, Dividing by $a^2 – ab$.

We know 2 is a different number than 1, and so, equality is not possible. Somewhere our argument has been invalid. Where? Look at the last step. Unknowingly, we have divided by 0. How? Look at $a^2 – ab$. This expression is equal to 0, whatever a and b may be, because we started with a = b which gives a^2= ab when multiplied by a and then $a^2 – ab = 0$.

Errors/mistakes/fallacies form integral part of learning Mathematics. It will help students if we keep this perspective in mind, when we try to examine students' work. One of the possibilities that students' work is incorrect because they have not applied procedural steps of solving problems.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

5) Explain the meaning of counter example with an illustration.

....

....

....

....

....

6) List a few fallacies that children of your class possess about the concept of measurement of area and volume.

....

....

....

....

....

7) How do children develop hypotheses?

....

....

....

....

....

1.5 CREATIVE THINKING IN MATHEMATICS

Usually, in Mathematics classrooms, teachers do ask questions that have only one correct answer. How do students arrive at answers? Here children utilize their previous experiences thinking that the same process will work for the question raised by the teacher. Without any difficulty, students will succeed in finding the answer. In Mathematics, the thinking that helps students tackle problems having one correct answer is termed as inductive thinking. You would agree that convergent thinking does not help children much to develop their cognitive abilities. In a way we are restricting the thinking styles of children. Being teachers, instead of limiting their thinking, we should organise situations that help children think in multiple ways. This would help children in divergent thinking. The opposite of convergent thinking is divergent thinking, which leads to development of creative responses called creativity.

Creative thinking has a major role in mathematical reasoning. It is creative thinking that has led to the invention of new ideas and knowledge, not only in

Mathematics, but also in other subject areas. What is creative thinking? "Creative thinking involves being able to break away from routines and stereotype methods to think flexibly, and to generate new ideas and approaches to problems. The opposite of creativity is rigidity and fixation" (Haylock, 2010).The following are the major components of creativity:

- **Divergent Thinking:** Engagement with different possibilities and not just proceeding on a predefined path
- **Fluency:** Being able to decide the mathematical process to be followed as quickly as possible
- **Flexibility:** Having courage to change the process without getting bogged down
- **Originality:** Ability to think independently and not just repeat the steps shown by others
- **Appropriateness:** Follow paths which match with the conditions of the problem and not just follow any steps and hope to fit them somehow or the other.

How do we develop creative thinking skills in children? For a teacher, such a question seems to be incomplete. It is not only students who should think creatively, but also teachers have to think creatively in organising teaching learning activities. Below are given a few methods that you can think of in order to inculcate the skill of creative thinking in your students:

a) Make your teaching more meaningful. Don't allow children to act as passive listeners and ensure active participation of children in learning process

b) Start teaching with concrete concept and slowly build up the difficulty level. Try to explain real world problems with simple examples.

c) Engage students in problem-solving situations by providing problems that have multiple solutions.

d) Motivate students to ask questions and pose problems themselves. Promote self engagement of children in learning activities.

e) Ask children to observe environment, people, situations etc., and thereby, develop the habit of observation.

f) Exposure to various learning strategies such as assignments, projects, etc.

g) Try to provide problems that involve divergent thinking and show the ways of reaching answers in multiple ways.

h) Do reward students who come up with innovative ideas.

i) Develop the quality of persistence. Do not get bored up while solving problems.

j) Develop a risk taking ability and engage in creative activities.

k) Avoid rigid and fixed pattern of solving problems.

l) Promote interaction with teachers and among peers.

It is equally important that teachers understand the ways of developing creativity and at the same time trying to organise learning activities that promote creativity. Let us see, how Mrs. Ancy made her students develop creative thinking. She asked the following question to her ninth standard students:

Question: What is the similarity in the expressions $5x^2$ and $70y^3$? Can you enlist the points and provide evidence for the argument?

The problem given above has multiple solutions. The children were unaware of the fact that the question has multiple solutions. They were thinking that the question is of the same kind as the teacher used to give them. But, when students started attempting to tackle the problem, they could realise that the problem had multiple solutions. Some of the answers given by students are given below:

- Both are polynomials.
- Both contain whole numbers.
- Both have constants.
- Both the constants are factors of 5.
- Both involve exponents.
- Both have coefficients.

Here, children get opportunity to think freely and originally. We should encourage creativity by acknowledging different solutions, evaluating them for elegance and efficiency.

Check Your Progress

Note: a) Write your answer in the space given below.

b) Compare your answer with those given at the end of the Unit.

8) How will you promote creativity among children of your class?

....

....

....

....

....

1.6 LET US SUM UP

Mathematics is a core subject at school level and has got number of applications in daily life. Thus the teachers are expected to develop understanding about the nature of mathematics and the factors that would enable to treat mathematics as a discipline. Being the first unit of the block, we have discussed the concept and nature of Mathematics in detail. As children validate mathematical knowledge they make use of various processes such as developing hypothesis, formulating conjectures and generalisation, proving mathematical arguments and statements, etc. These are the key processes used in mathematical reasoning and validation of mathematical knowledge. In later portion of this unit, the various processes involved in mathematical reasoning have been extensively discussed citing examples pertaining to them. The unit ends with the discussion on the ways of developing skills of creativity among children.

1.7 UNIT END EXERCISES

1) Prepare a brief report on the development of Mathematics focussing on the contribution of Indian mathematicians.
2) Describe a learning activity that enhances the creativity of children in Mathematics?
3) How will you help your children to be familiar with the concept of conjecture, through an example? Carry out the activity in your classroom and record your observations of children.
4) Ask children of your class to generate hypotheses of their own and test the same. Prepare a report on the classroom activities organised.
5) As a Mathematics teacher, what are your observations about the involvement of children in your Mathematics classrooms?

1.8 ANSWERS TO CHECK YOUR PROGRESS

1) When we wake up in the morning, the ringing up of alarm makes use of the concept of time. Time is a concept taught in Mathematics.
2) In Physics, to find the velocity of an object, the mathematical equation (y=displacement /time) is used. Thereafter, values are placed for displacement and time and the mathematical operation of division is carried out.
3) Refer to section 1.4.1
4) Generalisation, conjecture, counter example, hypothesis and proof.
5) A counter example is a specific case that demonstrates that generalisation is not valid. Give example yourself.
6) Do it yourself.
7) Check the common characteristics among cases, and a common statement (hypothesis) is generated.
8) Refer to section 1.5. A few of the tactics are provided to solve problems having multiple questions, Give assignment and projects, etc.

1.9 REFERENCES AND SUGGESTED READINGS

- Beckmann, C. E., Thompson, D. R. and Rubenstein, R. N. (2010).*Teaching and Learning High School Mathematics*. New Jersey: John Wiley and Sons Inc.
- Courant,R; Robbins, H. and Stewart, I. (1996). What is mathematics?: An elementary approach to ideas and Methods. London: Oxford University Press.
- DEP-SSA.(2009). *Teaching of Mathematics at Upper Primary Level (Vol I and II)*.New Delhi: Distance Education Programme-Sarva Shiksha Abhiyan.
- Haylock, Derek. (2010).*Mathematics Explained for Primary Teachers (fourth edition)*.New Delhi: Sage Publications India Pvt. Ltd.

- IGNOU (2008).*AMT-01Teaching of Primary School Mathematics*,AMT-01,Block 1-5,SLM.New Delhi: IGNOU.
- IGNOU (2010).*LMT-01 Learning Mathematics*, LMT-01, Block 1-6, SLM. New Delhi: IGNOU.
- IGNOU (2012).BES-009 *Teaching of Mathematics for the Primary School Child*, Block 1-4,SLM.New Delhi: IGNOU.
- NCERT (2005). *National Curriculum Framework-2005*. New Delhi: NCERT
- NCERT (2006). *Position Paper: National Focus Group on Teaching of Mathematics.* New Delhi: NCERT.
- NCERT (2008).*Source Book on Assessment for Classes I-V*. New Delhi: NCERT.
- NCTE (2009). *National Curriculum Framework for Teacher Education.* New Delhi: National Council for Teacher Education.
- NCERT (2012). Pedagogy of Mathematics, Textbook for Two Year B.Ed Course, New Delhi: NCERT.
- MHRD (1986). National Policy on Education. MHRD, Govt. of India: New Delhi.
- Anice,J. (2005). Teaching of Mathematics. Hyderabad: Neelkamal Publication Pvt. Ltd.
- http://www.slideshare.net/drangelrathnabai/nature-characteristics-and-definition-of-maths retrieved on 19/01/2016

UNIT 2 AIMS AND OBJECTIVES OF TEACHING-LEARNING MATHEMATICS

Structure

2.1 INTRODUCTION

Whatever may be the subject, any teaching-leaning is organised keeping in view the aims that are broad and comprehensive in nature. The aims are achievable through the realization of objectives that are narrow and specific. Teaching-learning of mathematics also takes account of well defined objectives. Developing mathematical knowledge, understanding various mathematical terms, widening appreciation towards Mathematics, and expanding skill of applying acquired knowledge in everyday activities are some among them. This Unit explains the most critical objectives expected to be achieved by teaching-learning of Mathematics namely; mathematisation of children's mind, development of reasoning power and visualisation, expansion of the skill of problem solving, increasing the power of critical thinking and so on. Also, Mathematics and its relation with life and its integration with other subjects are also discussed.

2.2 OBJECTIVES

After completing this unit, you will be able to:

- identify the need for establishing aims and objectives of teaching mathematics;
- state aims and objectives of teaching mathematics upper primary and secondary level;

- understand meaning of mathematisation and ways of mathematisation of mind ;
- distinguish between inductive and deductive reasoning;
- explain the use of visualisation in validating mathematical knowledge;
- describe steps involved in problem solving;
- comprehend the skills that help in solving mathematical problems;
- suggest activities to develop critical thinking in Mathematics;
- explain relationship between Mathematics and experiences in life; and
- discuss the relationships between Mathematics with other subjects.

2.3 AIMS AND OBJECTIVES OF TECAHING MATHEMATICS

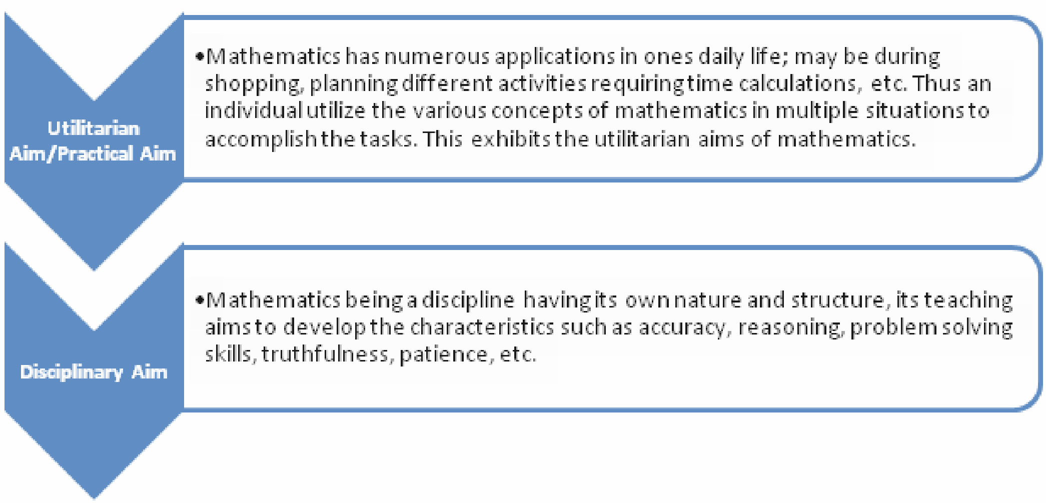
Teachers need to well aware about the different aims and objectives of Mathematics teaching. A teacher who has sufficient knowledge on these aspects would be able to develop various skills among his/her students. Let us discuss the aims and objectives of teaching Mathematics.

2.3.1 Aims of Teaching Mathematics

First of all let us try to understand the difference between goal, aim and objective of teaching Mathematics. The term 'aims of teaching Mathematics', stands for the goal or broad purpose that needs to be fulfilled by the teaching of that subject in the general scheme of education. Goals and Aims are like ideals, and their attainment needs long term planning. Therefore, they are divided into some definite and workable units named as objectives. The specific objectives are those short term, immediate goals and purpose that may be achieved within the specified classroom transactions" (NCERT, 2012). Thus we can conclude that aim is more broad, comprehensive and general in nature while objectives are means to achieve the aim.

According to NCGF-2005, the main goal of Mathematics education in school is the mathematization of minds of children.

What are the aims of teaching Mathematics? A brief description of the general aims of teaching Mathematics is given in Figure 2.1 .



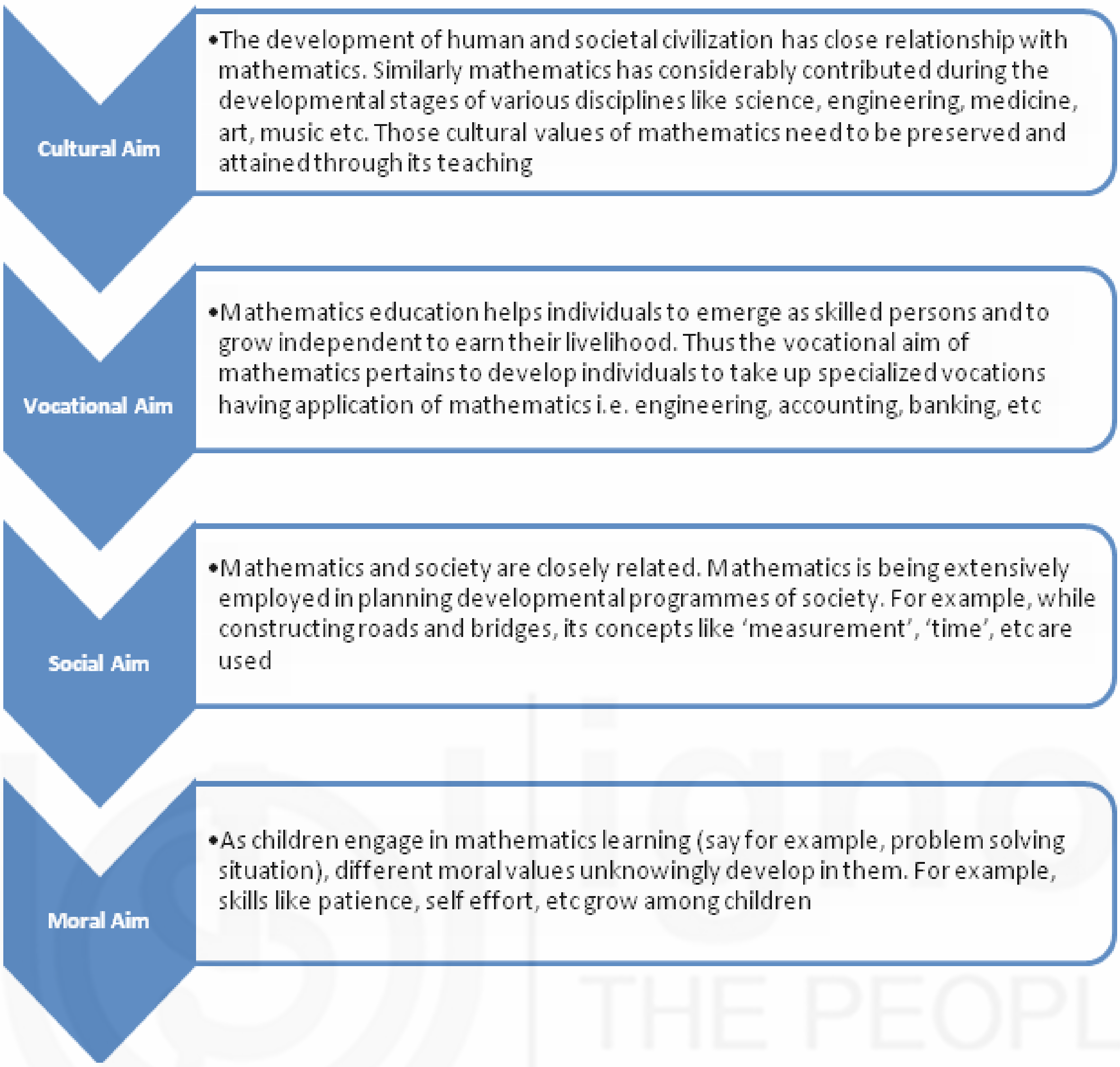


Fig. 2.1: General Aims of Teaching Mathematics

2.3.2 Objectives of Teaching Mathematics

We understood that aims are ideal general statements that are broad and comprehensive. Also aims are not definite and clear and require long term planning to achieve. In such a case, we move on to a more clear, achievable and workable units called objectives. Objectives are definite, clear, narrow, specific and can be attained in a short duration.

There are two types of objectives of teaching: general and specific. General educational objectives are broad and related to school and educational system.

Following are the general objectives of teaching Mathematics at secondary level.

The students will be able to:

- Acquire knowledge of facts, concepts, theories, laws, principles, proofs of Mathematics;
- Develop the ability to communicate mathematical ideas with precision and accuracy;

- Develop inertest and positive attitude towards Mathematics;
- Apply mathematical knowledge to solve real life problems;
- Develop the skill to use algorithms in problems solving;
- Appreciate the contributions of mathematicians;
- Develop mastery of algebraic skills, drawing skills, deducing interpretations, finding patterns, making connections, analyse, organise data, reasoning, critical thinking, etc.

Specific objectives are short term immediate goals or purposes that may be achieved through classroom instructional/educational process. Thus we may call such a objective as **educational objective/instructional objective**. In the case of classroom instructions, teacher is concerned about bringing changes in the behaviour of learners and we call that specific objective as **behavioural objective**. Behavioural objectives are the objectives that written in behavioural terms. It explains the change in state of behaviour of an individual on completion of a learning activity.

As we discussed earlier, educational processes aspire for bringing behavioural changes in an individual. Bloom has classified the change in behaviour in three domains or categories namely; cognitive domain, affective domain and psychomotor domain. Bloom (1956) had organised various educational objectives in a hierarchical order and we call it as **Bloom's Taxonomy of Educational Objectives.** The educational objectives falling in each domain is hierarchically placed in ascending order of complexity. Even though, many taxonomies are available the most acceptable and widely used one is Bloom (revised) and others for cognitive domain; by Krathwohl for affective domain and Dave for psychomotor domain.

Cognitive Domain **Bloom(1956)**	**Affective Domain** **Krathwohl, Bloom, Masia (1973)**	**Psychomotor Domain** **Dave(1975)**
Knowledge	Receiving	Imitation
Comprehension	Responding	Manipulation
Application	Valuing	Precision
Analysis	Organization	Articulation
Synthesis	Internalizing Values (Characterization)	Naturalization
Evaluation		

Table 2.1: Taxonomy of Educational Objectives

In the year 2001, Anderson and Krathwohl modified the objectives belonging to the cognitive domain of Bloom's taxonomy. The pictorial representation of Anderson and Krathwohl Taxonomy 2001 is given in Figure 2.2.

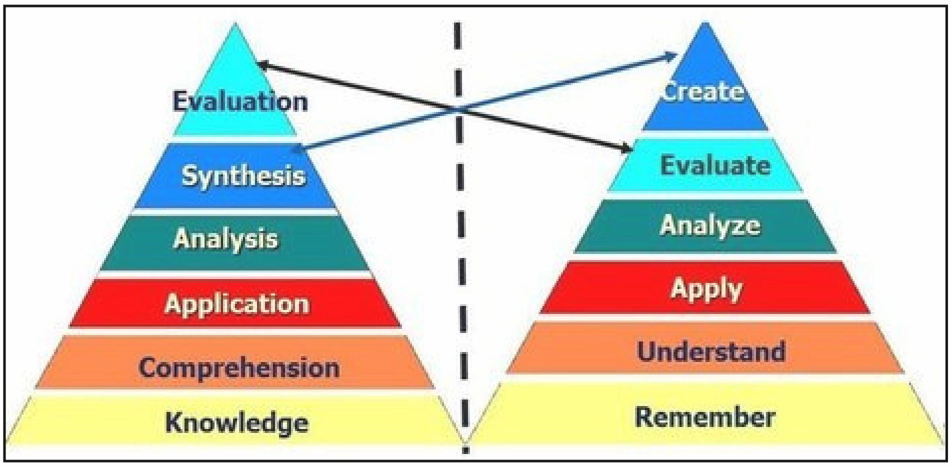


Fig. 2.2: Anderson and Krathwohl Taxonomy 2001

(**Source:** http://www.scoop.it/t/cognitive-rigor?page=2)

Keeping in view the context of objectives of teaching Mathematics let us briefly discuss the taxonomy of educational objectives. It is the duty of the teacher to first locate different learning (specific) objectives pertaining to the topic/concept that he/she is going to teach. After deciding the learning objective, the different learning experiences/activities and assessment mechanisms are chosen. Thus the learning objectives are written with the help of action verbs that are clear and specific. The action verbs give the direction to the teacher about 'what the children will do' or 'what the learners are expected to do' after completion of the learning activity. Thus, with each learning objective, action verbs and specifications are associated. Now let us discuss the learning objectives belonging to different domains.

Cognitive Domain: Cognitive means 'knowledge'. The levels falling under cognitive domain are as follows:

Remember: 'Remembering' is the lowest level objective of cognitive domain. It refers to the ability to recall information of facts, concepts, theories, laws, patterns, structures, generalizations, etc. A child who has the ability to recall mathematical information would be able to proceed to acquire highest learning.

Examples: recalling the definition of rectangle, formula for finding the area of rectangle.

Understand: It is the next higher level of cognitive domain. Understanding helps learners to correlate, connect and develop meaning from new material.

Example: describing the method of calculating area of rectangle.

Apply: The learner is able to apply different facts, concepts, theories, laws or principles in new situation. Application subsumes both knowledge and understanding.

Example: applying knowledge of calculating area of rectangle to find the area of own house of the learner.

Analyze: Analysis is the breaking down of a complex situation in to different parts/elements. At such a stage, the leaner will be able to locate the elements, differentiate, recognize relationship, and identify patterns pertaining to a situation.

Example: identify the causes of splitting the following figure into different parts for calculating its total area.

Evaluate: Evaluation represents the learner's ability to formulate hypothesis, critique, and judge a material, situation or method against the standard, which may be internal or external to it. Evaluation is the most complex mental process belonging to cognitive domain.

Example: justifying the need for constructing 'rooms' in rectangular/square shapes.

Create: As the word implies creating stands for collecting information, designing and putting elements together to construct a new pattern or develop theory out of it or to build up understanding of the concept in detail.

Example: making a rectangular shaped house using thermocol sheet.

Affective Domain: It deals with the emotional aspects of the learner. The various emotional states of an individual like different feelings, motivation, interest, attitude, values, appreciation, etc fall under affective domain. Similar to cognitive domain, the learning objectives of affective domain are hierarchically ordered i.e. from simple to complex. The learning objectives pertaining to affective domain are given below:

Receiving: Receiving is the lowest level objective of affective domain. Receiving refers to the learner's ability to listen and receive a situation, stimuli, phenomenon, information, etc. For example, listening to the teacher's lecture on the topic 'area of rectangle'.

Responding: In this stage of mental process, the leaner starts responding to different situations, information and stimuli. For example, asking teacher the difference between perimeter and area of the rectangle.

Valuing: Valuing involves increasing internalistaion of the worth or value a person attached to a particular object, phenomenon or behaviour (NCERT, 2012). For example, showing interest in solving problems related to rectangle.

Organistaion: Organistaion is the fourth level objective which brings together different values, resolving conflicts between them, starting to build an internally consistent and a unique value system or attitude (NCERT, 2012). For example, showing the attitude to solve mathematical problems by self.

Internalising Values (Characterisation): Characteristaion is the highest level objective in which the values and attitudes of an individual is attained to help to control his/her behaviour. The personal, social and emotional behaviour of an individual reflects his/her attainment of values. For example, while solving mathematical problems, maintaining patience till he/she reaches answer.

Psychomotor Domain: Psychomotor domain includes the ability to use body parts to accomplish tasks, neuro muscular movements and types of body actions. The psychomotor skills can be observed while playing, typing, stitching, etc. Psychomotor skills are developed through practice and are measured in terms of speed, precision, accuracy etc. The objectives belonging to psychomotor domain are given below:

Imitation: Imitation is the lowest level objective of psychomotor domain. At this stage, individual observes actions and are practiced/repeated/simulated at his/her mental level. Later, the individual performs those actions but with less precision. For example, constructing a rectangle by using matchsticks.

Manipulation: Manipulation involves listening to other's directions, selecting certain actions in preference to others and practicing those actions for accuracy and perfection. For example, listening to the teacher and build a rectangle as per the teacher's advice.

Precision: The third level objective of psychomotor domain implies the development of motor skills with exactness. At this stage, the control over actions helps him/her to develop required motor skills with precision. For example, assembling various objects, take measurements and then construct a rectangle with accurate measurements.

Articulation: Articulation involves control over multiple motor skills in a logical and systematic way which help individual to complete the desired action. High level coordination of various motor skills is developed at this stage. For example, to construct a rectangle, firstly, the items needed are collected, measurements are taken, the procedures are followed and jotted down then finally the rectangle is made in a sequential order.

Naturalistaion: The highest level of psychomotor domain, at this level, motor skills and coordination of movements becomes the reflex actions/mechanical. While performing any action, the individual naturally performs with precision and accuracy.

For example, when children are asked to construct a rectangle, automatically the materials required comes to their mind, and they succeeds in constructing it.

Formulation of objective will guide you in your teaching learning process and in turn help children to achieve the desired learning outcomes.

Now we may discuss few of the general aim and objectives in detail like mathematisation of mind of children, enhancement of reasoning power and visualization, developing problem solving skills and critical thinking in Mathematics.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

1) What are the general aims of teaching mathematics? Write any three aims.

……………………………………………………………………………

……………………………………………………………………………

……………………………………………………………………………

……………………………………………………………………………

2) What are the general objectives of teaching Mathematics at secondary level? Write any three objectives.

……………………………………………………………………………

……………………………………………………………………………

……………………………………………………………………………

……………………………………………………………………………

……………………………………………………………………………

……………………………………………………………………………

3) Write any four specific objectives for the unit 'Probability'.

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

2.4 MATHEMATISATION OF MINDS OF CHILDREN

Mathematics is a subject that has relevance to day-to-day life. For example, a child buys a notebook costing ₹ 30 and gives the shopkeeper a ₹ 50 note. Here, the shopkeeper returns ₹20 to the child. What mathematical thinking is involved here? The shopkeeper has used the basic mathematical operation 'subtraction', while returning ₹20 to the child. This way, each individual utilizes Mathematics in his/her day-to-day life. How is it possible for an individual to think in mathematical terms? Teachers have great role in this regard. As we know, apart from teaching mathematical facts, teachers should help children to develop the skill of utilizing Mathematics and seeing things in mathematical way. Mathematics is not for acquiring good grades, rather, it is for helping children to be able to deduce mathematical interpretations of the circumstances and judge through mathematical thinking. ***Mathematisation refers to the act of interpreting or expressing mathematically, or the state of being considered or explained mathematically***. In other words mathematisation is nothing but converting a given situation in a more specific way using numerals, symbols and mathematical facts satisfying the given conditions with the help of logic. Logic is the back bone towards mathematisation of any given situation.

The teaching- learning of mathematics stress much on developing the skill of mathematisation. It is expected that, children should expand the horizon of cognition by incorporating abilities that help them manage situations mathematically. Here, children should make use of the knowledge, facts and principles of Mathematics to arrive at a judgement. This is referred to as mathematisation of mind. A child, who has developed the ability to think in mathematical way, is capable of tackling and finding solutions to all problems that she/he encounters. So, it is a must on the part of teachers that they should organise learning activities that emphasise the skill of development of mathematisation. Let us look at how Mr. Jagat has organised an activity to achieve this aim.

Activity 1

Mr. Jagat is teaching his seventh class students. He divides the class into different groups. He told, "Today we are going to play a game. It is a simple game. Two students will act as shop keepers. They will sell materials such as notebooks, pens, lunch boxes, etc. The rest of the students should buy materials from them. You should buy materials for yourself and your brothers/sisters. Finally, you should prepare a chart containing details of all materials that you have purchased. Also, you should write one more thing i.e. the thinking process that you have followed in buying the particular material". After explaining the procedure of the game, Mr. Jagat started observing the children. A few minutes later, children came up with their results. The result given by one of the students, Radhika, is given below:

S.No	Material Purchased	Quantity
1	Notebook	20
2	Pen	10
3	Pencil	15
4	Geometry Box	3
5	School Bag	3
6	Textbook	5

Process Followed:

- I have ***one*** sister and ***one*** brother. So I thought that they also ***need notebooks for one year***. So, I purchased 20 notebooks. I know, we ***require more than 20 notebooks***, but I ***didn't have money to buy more than 20***. So, I ***limited my purchase to 20.***
- The ***cost is very low*** for pens. So, I thought, I ***would buy 10 pens*** so that we ***three can use them for the entire year.***
- Pencil ***doesn't last long***. We ***used to buy 4/5 pencils every year***. Also, the ***price is very less for*** it. So I purchased 15 pencils.
- The ***shop sells good quality geometry box and school bag***. So, I thought three pieces of each is enough for us.
- Lastly I purchased text books because I ***was left with ₹ 100 only***. Then I ***calculated that, ₹ 80 are required to purchase 5 textbooks***. So, I purchased them. I ***didn't have the money*** to purchase textbooks for others.

What can we conclude from this activity? Here, the teacher was trying to introduce the chapter on 'statistics'. He creatively organised an activity and finally used the same data to introduce the concept of frequency. In the Box given above, the letters in bold ittallic words show childrens' ways of mathematical thinking. Here, the students confronted a problem that they should buy school materials for themselves and their siblings. Also, they are left with a particular amount. In such a situation, they had used the mathematical thinking to find a solution (here the purchase of materials with the money they have in hand). As a teacher, you may organise innovative activities that help children to develop knowledge in the subject as well as chances to build their mathematical thinking.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

4) What do you mean by mathematisation?

....

....

....

....

5) Suggest an activity that would enhance the skill of mathematical thinking of children.

....

....

....

....

2.5 ENHANCEMENT OF REASONING POWER AND VISUALIZATION

It is reminded that we are discussing the major aims of Mathematics teaching. The next aim is concerned with development reasoning power and generalisation in which a part of the later has been discussed in the previous unit. Reasoning and visualisation go hand in hand and are key processes in validating mathematical knowledge. **Mathematical reasoning enables children to arrive at solutions/judgements/conclusions after manipulating the facts involved in the problems**. To solve problems, children evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how these solutions can be applied. Mathematical reasoning involves two processes, namely: **inductive reasoning and deductive reasoning**. Children knowingly or unknowingly use both inductive and deductive reasoning in various situations. **Inductive reasoning is the process of observing data, recognizing patterns, formulating hypothesis, and making inferences.** It is fundamental to learning of Mathematics**.** Inductive reasoning, not only develops problem solving skills, but also facilitates learning of Mathematics.

When you draw conclusions on the basis of observations from experience, you use inductive reasoning. Mathematicians use inductive reasoning to find out the true solutions. It proceeds from concrete to abstract, from particular to general and from examples to general formulae. It helps in developing facts, concepts, principles, rules, definitions and generalisations, and basic other elements of Mathematics. In inductive reasoning, we establish a relation by experimenting with numerous examples and wish that it would be the same for similar cases, and hence, it helps to discover new body of knowledge. Mathematics curriculum expects children to develop the skill of reasoning to seek answers for mathematical problems. An activity that can be organised in classrooms to enable children to develop the skill of inductive reasoning is as follows:

Activity 2

The activity is organised to establish a formula to compute the simple interest. Here, the teacher, Mr. Akshay has worked a series of steps to make his children develop the formula of simple interest. The steps are given below:

Step 1: In the first step, Mr. Akshay presented many examples related to the calculation of simple interest (S.I.). The students use one unitary method to calculate S.I.

Example 1: Find the simple interest on ₹1000 at the rate of 4% per annum for 3 years.

Solution

S.I. of ₹100 for 1 year = ₹ 4

S.I. of ₹1 for 1 year = ₹ $\frac{4}{100}$

S.I. of ₹1000 for 1 year = ₹ $\frac{4 \times 1000}{100}$

S.I. of ₹1000 for 3 year = ₹ $\frac{4 \times 1000 \times 3}{100}$ = ₹120

Example 2: Find the S.I. of ₹700 at the rate of 4% per annum for 3 years.

Solution

S.I. of ₹100 for 1 year = ₹ 4

S.I. of ₹1 for 1 year = ₹ $\frac{4}{100}$

S.I. of ₹700 for 1 year = ₹ $\frac{4 \times 700}{100}$

S.I. of ₹700 for 3 year = ₹ $\frac{4 \times 700 \times 3}{100}$ = ₹84

Step 2: Here, the teacher asks questions based on the examples worked out above. These are as follows:

Teacher : What is the S.I. in example 1?

Students : ₹120.

Teacher : What is the S.I. in example 2?

Students : ₹84.

Step 3: Next, the teacher asks the students to observe the above examples. Then, he asks the students establish a relation to find the S.I. At this stage, teacher helps children in forming the formula. With the help of the teacher, children developed the formula to find S.I. and are as follows:

From example 1, S.I. for 3 years = $\frac{4 \times 1000 \times 3}{100}$

From example 2, S.I. for 3 years = $\frac{4 \times 700 \times 3}{100}$

Therefore, S.I = $\frac{P \times R \times T}{100}$ (Where; P=Principal, R=Rate and T=Term)

Step 4: In the final steps, the teacher provides similar problems and the students are asked to solve them by using the derived formula.

In the above discussion, a formula is derived by looking at a few examples. This way of reasoning is called as inductive reasoning. As a teacher, it is our duty to provide problems that may result in derivation of facts, concepts and rules of Mathematics. Now, let us discuss deductive reasoning. **Deductive reasoning is exactly opposite to inductive reasoning in which the individual proceeds from general to particular, from abstract to concrete, and from formula to examples.** Generally, deductive reasoning finds application at the higher classes; even then at times secondary teachers utilize it for teaching-learning. Here, the child has to memorise numerous formulas and rules which are used to solve problems. For example, the formula S.I. = [P X R X T]/100 is used by children to find simple interest. At this stage, children will be given a problem and the formula. The only role of the children is to apply the given data and generate result.

It is hoped that you have understood the concepts of inductive and deductive reasoning. Now, let us discuss, why visualisation is important in Mathematics and how children use it to generate ideas. Listen to a classroom interaction given below:

Teacher : Hello students, listen to me. I am giving you a problem. Try to find solution on your own. O.K?

Students : Yes sir

Teacher : The question is A boy of 96 cm height is walking away from the base of a building at a speed of 1.1m/s. If the height of the building is 5.4m, find the length of boy's shadow after 5 seconds.

In such problems, what will children do first? They will try to make a mental image of the situation. In this case, they will visualise the building and boy. The figure is produced on the paper which would be similar to the following:

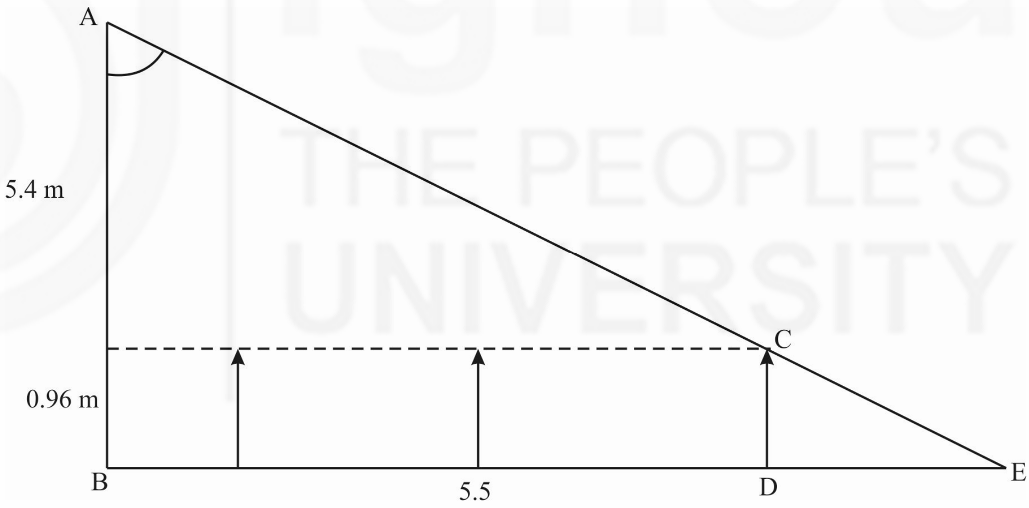


Fig. 2.3

After drawing the figure, the mathematical calculation is tried as follows:

We have to measure the distance DE= x metres

Here BD=1.1 × 5 = 5.5

In ΔABE and ΔCDE

∠B = ∠D (Both are 90^0 as building and boy are standing vertical to ground)

and ∠E = ∠E (Same angle)

so ΔABE ~ ΔCDE (AA similarity criteria)

Therefore, BE/DE=AB/CD

i.e. $\frac{5.5+x}{x}=\frac{5.4}{0.96}$

or x = 1.18

Therefore, the shadow of the boy will be 1.18 m long after walking for 5 seconds. What conclusion do you draw about the classroom interaction? In Mathematics classroom, children do attend problems and their complexity differs. At times, the problems will not be solved directly by applying mathematical equations. Even, children find it difficult to figure out mathematical equation that suits a particular problem. In such cases, children have to develop a mental image of the situation to make sense out of it. The mental image (model) represents the mathematical situation. Even children may visualise like "what will happen if I represent it like....?'. So **visualisation is not just pictures or diagrams, instead, it represents the mental image (model/sketch) of any situation**.

In general terms, visualisation is formation of mental image of something. For example, image of an elephant, school, parliament etc. But in Mathematics, children develop the mental image of mathematical situations and facts. For example, mental image of a cube, bar graph and so on. Why visualisation is important in Mathematics classrooms? How can teachers develop the visualising skills among children? These questions should be considered while designing learning activities. Visualisation helps children understand a complex problem. It helps him to cut down the components to digestible elements so that a connection could be made among the data that appear in the problem. The complex situations can be easily represented as images that help to understand and develop a plan to work out the solution. Generally, visualisations serve three purposes, namely;

- To acts as a support to step into the problem;
- To model the present problem/situation; and
- To plan the strategies for solving the problem.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answer with those given at the end of the Unit.

6) Differentiate between inductive and deductive reasoning.

....

....

....

....

....

....

....

....

....

2.6 DEVELOPING PROBLEM SOLVING SKILLS

In Mathematics classrooms we often find that students are solving problems. Students are able to respond better when they are given direct question where a particular method is to be applied as well as which method is to be applied. However when they are given questions in which they have to decide upon which method to apply, then they become uncomfortable. Most students face difficulty in finding solutions to word problems. This happens in the classroom because students have not developed the skill of problem solving.

Problem solving is a process and comprises of a sequence of steps: ***Understand, Think, Try, and Look Back.*** Let us discuss these steps of problem solving.

First is to Understand the Problem-Before you can solve a problem you must first understand it. Read and re-read the problem carefully to find what the question is being asked to you to solve. Separate what is given and what is to be found. Write down the information given as points. Write a statement about what is to be found.

Then Think --Once you have understood the problem, look for strategies and tools. Here your previous knowledge will be of great help. So try to remember if you have come across similar situation earlier and recall what you did then. If it turns out to be unfamiliar problem, try to search through your knowledge of Mathematics and identify what could help you in this situation. You can go through your reading material and books. You can work backwards which means look at what is to be found and try to think what can make it possible.

After that Try It–If it is a familiar problem, try if the previous method used works or any modification of the same will work. If it unfamiliar problem try some method which you may have already decided are relevant.

Look Back--Once you've tried it and found an answer, go back to the problem and see if you've really answered the question. Sometimes it's easy to overlook something. If you missed something then check your plan and try the problem again.

As students follow these steps which can be used to arrive at specific strategies. However, selection of the strategy is based on the nature of the problem that they are trying to solve. Some strategies are given below:

- look for a pattern
- draw a diagram or picture
- make a simpler but similar problem
- work backwards
- use a formula
- guess and check
- make and state assumptions
- consider alternative strategies and/or blend strategies
- monitor progress and revise, as necessary.

You as a teacher can help children to select strategy to solve the problem. You can give different types of problems in your class to children where problem solving skill is required. Let see what type of problem you can give to your students.

Example: Raju hired a taxi from his home to the city, which is at a distance of 10 km. During the journey he came to know that taxi charge in the city consists of a fixed charge along with charge for the distance travelled. He paid Rs 105 for the journey. From the city he hired another taxi for travelling up to a hospital, which is 8 km away from the city. He paid Rs 85 to the driver. What are the fixed charges and the charge per km in that city?

Step 1: Understand the problem: Without understanding or comprehending the problem, the student may not be able to solve it correctly. During this process, students may ask themselves some questions like:

- ✓ What is to be found out or shown?
- ✓ What data or information are given?
- ✓ Do I understand all the facts and terms given?
- ✓ Are given data or information sufficient for reaching the solution?, If not, what more information is required?
- ✓ Are there any irrelevant information?
- ✓ How can I restate the problem in a better way?
- ✓ Am I required to draw a picture or diagram?, If yes, then how can I draw it?

Students can pose these types of questions for clearly understanding the problems. All questions may not be applicable to all types of problems. Depending upon the problem, students can frame appropriate questions.In the above example students can ask questions such as:

Question	**Expected answer**
What is to be found in the problem?	**The fixed charge and charge per km for the hired taxi.**
What information is given in the problem?	**Taxi charge consists of a fixed charge + charge for distance travelled.** **Taxi charge paid for 10 km travel is Rs 105.** **Taxi charge paid for 8 km travel is Rs 85.**
Are the data given sufficient for arriving at solution?	**Yes.**

By answering such type of questions students can move to the next step.

Step 2: Then think and Devise a Plan: This step explains the 'how' aspect of finding the solution to the problem. Here, students need to think and device an appropriate strategy for solving the problem. Based on their previous experience, students can come up with a concrete plan. They can initiate this step on the basis of answers to the earlier queries. If the data given are sufficient, and if students know what to find out, they can link those two informations and can raise another question, "how can I find the desired solution with the help of given data?"

In the above example, students can realise through questioning that, they are expected to find the fixed charge and charge per km for the hired taxi (two variables/unknown).

The data given are taxi charge paid for a distance of 10 km and for a distance of 8 km. This amount consists of fixed charge and charges per km, both are unknown. If these unknowns can be assumed to be x, and y respectively, these statements can be converted into two equations in two unknown, and subsequently, procedure for solving them can be utilised.

Students thinking process can be reproduced as follows:

Taxi fare = Fixed charge + Charge per km × Distance travelled

For charge paid for 10km travel = 105

i. e, Fixed charge + 10x Charge per km = 105

Similarly, fixed charge + 8x Charge per km = 85

Assume that, the fixed charge is 'x' and charge per km is 'y' the above two can be represented in terms of two equations

$x + 10y = 105$ and $x + 8y = 85$

Based on this analysis, the student can finalise as to which strategy can be used for solving the problem

Step 3: Carry out the plan*:* After devising a suitable plan for solving the problem, the students can move to execute the plan.At this stage, students have to solve the above mathematical equations in two unknown using any strategy for solving simultaneous equations in two unknown.

They can go for either substitution method or elimination method. Suppose they are using substitution method, they will solve the equations in the following way

$x + 10y = 105$……………(1)

$x + 8y = 85$……………..(2)

From (1), $x = 105 – 10y$………(3)

Substituting the above value of x from (3), in equation (2)

We get, $105 – 10y + 8y = 85$

or $105 - 2y = 85$

or $-2y = 85 - 105$

or $-2y = -20$

or $y = 10$

From (3), $x = 105 – 10 \times 10$

or $x = 105 – 100$

or $x = 5$

Therefore, fixed charge is Rs 5, and charge per km is Rs 10.

Step 4: Look back: After finding the solution, it is important to verify that solution. Students may be encouraged to check not only the result, but also the various steps, procedure and calculation also. They may be encouraged to verify the solution by answering the questions like 'Is the answer reasonable?', 'Is there another method of solution that will easily verify the answer?' ,'Does the answer fit with given data?', etc

For example, in the above problem:

Taxi charge for travelling a distance of 10 km is 105

ie, fixed charge + charge for 10 km = 105

therefore, $5 + 10\times10 = 105$

ie, $5 + 100 = 105$

or, $105 = 105$

Similarly, taxi charge for travelling a distance of 8 km is 85.

or, fixed charge + charge for 8 km = 85

or, $5 + 8\times10 = 85$

or, $5 + 80 = 85$

or, $85 = 85$

One way to inculcate a positive attitude towards Mathematics and develop interest in the subjects among students is to provide them with various types of problems and help them to solve them independently as far as possible. The satisfaction of the experience of success in solving the problem will definitely enhance their confidence, and in turn, positive attitude towards Mathematics. If we are able to provide problems or tasks that encourage reflection and communication, and those are selected from students' real life situations, they will take the subject seriously and will appreciate the value of learning Mathematics.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

7) Discuss the steps involved in problem solving with examples.

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

8) According to your experience, what are the steps children use to arrive at solutions to mathematical problems?

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

…………………………………………………………………………

2.7 DEVELOPMENT OF CRITICAL THINKING IN MATHEMATICS

Critical thinking is defined as meaningful, unbiased decisions or judgments based on the use of interpretation, analysis, evaluation, inferences, and explanations of information as it relates to the evidence applied to a specific discipline. Critical thinking is the way of deciding whether our claim or judgment is true or not. We make reasoned judgments or decisions about what we think or do. So, forming judgments and decisions based on one's own experience or claiming one's own decision true or false with supporting proofs is called critical thinking. A child who thinks critically will himself/herself ask the following questions when he/she faces a perplexing situation/problem (may be a mathematical problem):

- What is the problem? How can I find solution to it?
- Can I solve it with the information I have? If not, what additional information is required?
- Can I solve this problem with the same formula as studied in the classroom?
- If I do it like this, what would be the answer?
- What other strategies and formulas will work here?
- Can this problem be solved by a single method or I should use some other method?

Today, the constructivist approach emphasise 'critical pedagogy' as a learning approach. Whatever the teacher taught is critically analyzed and questioned by the children that ultimately lead to the development of mathematical concepts by their won. As a teacher, how will you develop among children the skill of critical thinking? You may organise activities and let children find the solution by their own efforts. Also present mathematical problems that have multiple ways of solution. For example, teacher can ask the following question:

Question: Find the area of a sector of a circle with radius 5 cm which has an angle 40°. Also, find the area of the corresponding major sector (Use $\pi = 3.14$).

Solution

Given sector is OPAQ (See figure 2.4)

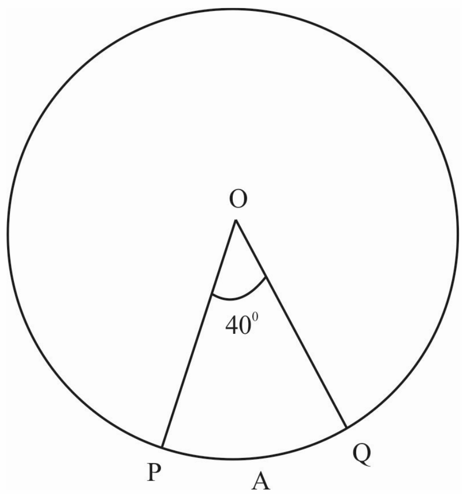


Fig. 2.4

Area of the sector = $\frac{\theta}{360} \times \pi r^2 = \frac{40 \times 3.14 \times 5^2}{360} = 8.72 cm^2$

Area of the corresponding major sector = πr^2 – Area of the sector OPAQ

$= 3.14 \times 5^2 - 8.72 = 69.78 cm^2$

The example given above suggests the way children will arrive at solution. But, we have an alternate method also. Usually, children do not show interest in attempting alternate methods, and thus, the skill of critical thinking is not developed. But children having the ability of thinking critically, then they experiment with new methods to arrive at solutions. The alternate method that helps to find solution to the above problem is given below:

Area of the major sector = $\left(360 - \frac{\theta}{360}\right) \times \pi r^2 = \left[360 - \frac{40}{360}\right] \times 3.14 \times 5^2$

$= 69.78 cm^2$.

In the example given above, the teacher has used a problem that has two ways of finding solution. As a teacher, you may try different methods. Apart from presenting mathematical problems, we can use the following strategies that will help children to develop critical thinking:

- Children may be presented with problems that do not have predetermined solutions and methods leading to answer.
- Provide children challenging questions and problems having multiple strategies to find solution.
- Children may be asked to find real life situations that have application of Mathematics.
- Encourage children to attempt open problems and questions with multiple solutions.
- Make children understand what they are supposed to do about problems.
- Make children focus on the type of problem thus, they are supposed to solve.
- Make children sure that they are working out the problems completely.
- Ask children to identify the mistakes committed while solving problems.
- Motivate children to attempt series of problems of varied types.

Check Your Progress

Note: a) Write your answer in the space given below.

b) Compare your answer with those given at the end of the Unit.

9) Suggest some ways to develop critical thinking among children.

....

....

....

....

....

2.8 INTEGRATION OF MATHEMATICS WITH OTHER SUBJECTS

In the previous section, we stated that "Mathematics is the science of all sciences and art of all arts". Is this statement valid? Why do we make such a statements? Does Mathematics have any relation with science and art? Yes of course! The presence of Mathematics can be seen anywhere including the subjects like sciences, biology, engineering, agriculture, philosophy, psychology, history, geography, drawing, arts, languages and commerce. So, Mathematics has correlation with all other subjects. Today integrated teaching is advocated as it helps children for easy assimilation of subject knowledge. Thus, while organizing learning experiences, teacher should help children to integrate Mathematics and concepts of other subjects. Let us first briefly discuss the correlation of Mathematics with other subjects and then ways of integrating it with other subjects.

Mathematics and Physics: Mathematics is closely associated with physics. In physics, we find many mathematical equations and formulas. For example, v=u+at, makes use of the concept of mathematical equation (equal sign). Similarly, the laws of motion, laws of levers, laws of reflection and refraction, laws of electric current, etc. use Mathematics. Light rays, levers, steam engines, telephones and communication devices, electromagnetic rays, electronic and semiconductor devices, etc. have the application of Mathematics.

Mathematics and Chemistry: All chemical combinations are governed by mathematical laws. The formation of a chemical compound is not possible without Mathematics. The basic unit of any substance, atom and the sub particles, of which atom is made up, obey mathematical laws. Chemical equations are controlled by mathematical principles. The huge amount of energy created by an atom is calculated using Mathematics.

Mathematics and Biology: Mathematical principles and facts are applied in all studies concerning botany and zoology. The caloric and nutritional values are calculated using Mathematics. The growth of plants and animals is measured; the respiration and transpiration of water in living bodies etc also use Mathematics. The study of nutrition, growth, maturation, compounds, mixtures, laws of chemical combination, molecular and atomic structure, chemical names and formulas etc all are based on mathematical laws.

Mathematics and Engineering: Mathematics is considered as the foundation of engineering. Surveying, levelling, building construction, construction of electronic and other mechanical devices, dams, bridges etc., use laws and principles of Mathematics.

Mathematics and Agriculture: In agriculture, the money involved, expenditure and income generated are calculated. Similarly, the time to start cultivation of crops and vegetables is analysed. Also, the measurement of plots for cultivation, production per unit area, cost of labour, seed price, etc are calculated using Mathematics.

Mathematics and Social Sciences: Economics employs mathematical principles and languages to interpret social phenomena. The share market operations, country's revenue statement, budget analysis, etc. use Mathematics.

In geography, the climatic changes, height of mountains, knowledge of rivers, population, moment of winds, area of earth, longitude and latitude, etc. are measured using Mathematics. In history, the historical developments are traced and analysed with Mathematics. Similarly, in philosophy, the basics of all subjects are formed with the help of Mathematics. In commerce, Mathematics is used in accounting and bank-related operations.

Mathematics and Psychology: All the psychological measurements related to human behaviour are collected using appropriate scale constructed using mathematical principles. Also, statistical methods use to organise, analyse and interpret psychological behaviour of any object/subject. Experimental psychology is based on mathematical computations.

Mathematics and Art and Drawing: The various branches of drawing like geometrical drawing, memory drawing, figure drawing, etc. employ Mathematics to produce beautiful colour combinations and pictures. A picture is attractive to eye when the proportion and ratio of colours are perfectly maintained. Even an artist, makes use of his/her mathematical knowledge before attempting to draw a picture on the canvas.

Mathematics and Language: It is language that helps Mathematics to express mathematical equations, laws and principles. Similarly, the medium of expression of any mathematical fact employs the use of language. The funny thing is that, language differs from place to place, but the mathematical idea remains constant irrespective of the language.

Having understood the relation between Mathematics and other subject areas, it is now the task of the teacher to integrate each subject with Mathematics in the teaching-learning process. How is it possible? Let us listen to a conversation. Here, Mr. Ramkishore is teaching the concept of 'arithmetic progression' to his tenth class students. Arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. The fixed number is called the common difference. How will a teacher introduce AP to his/her students? Normally, the teacher may say, "in the series of numbers 2, 4, 6,8,10, etc., each number is 2 more than the previous number. Such a list of numbers is called an AP". The teacher may give a few more examples. The children passively admit the concept and reproduce in the term tests. But, let us take note of the classroom interaction of Mr. Ramkishore.

Ramkishore : Hello children! how are you? Have you done yesterday's home assignment? Let me check it.

Students : Yes sir.

For a while, Ramkishore gets engaged in evaluating the home assignments. After checking the home assignments, he continued:

Ramkishore : Let me tell you an incident. The previous day, I went to a shop that sells pesticides. Do you know about pesticides?

Some students said 'yes' while a major group were unaware of pesticides. Then, teacher continued:

Ramkishore : I will tell you. Pesticides are substances which are used to destroy pests.

Ramkishore continued to talk about pesticides. He explained different kinds of pesticides, where they are found, the techniques to use pesticides, and so on. Then, he started narrating the incident;

Ramkishore : So, where did I stop? Aah.. yes. yes. I went to the shop. Then for ten minutes I watched the people buying pesticides. Then what I found was that a man purchased 1 kg pesticides, then the other man purchased 2 kg (he may be having more cultivation), a third man 3kg, fourth man, 4kg and so on. Now, my question is can you tell, the quantity of pesticides purchased in order?

Students : It is very easy sir. 1 kg, 2kg, 3kg, 4 kg, 5kg.....

Ramkishore : let me repeat, 1,2,3,4, 5, isn't it?(he write the numbers on the board)

Students : Yes Sir.

Ramkishore : Now, my next question. Do you find any relation in the numbers given above?

The discussion prolonged till the conclusion, Mr. Ramkishore introduced the concept of AP. In this example, Mr. Ramkishore has tried to correlate Mathematics with the subject of agriculture. In the process, he discussed the different aspects of the concept 'pesticide', a topic of agriculture. In a similar way, Mathematics can be correlated with other subjects.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

10) Why is integration important in Mathematics teaching?

....

....

....

....

....

11) How will you teach Mathematics by integrating it within concepts biology?

....

....

....

....

....

2.9 LET US SUM UP

Aims are broad in nature while objectives are the means to achieve it. Mathematics teaching enables to develop among children objectives like critical thinking, ability to think in mathematical terms, skill of problem solving, development of reasoning power and visualisation, etc. In this unit, we have extensively discussed the broad aims and objectives of Mathematics teaching at secondary level with adequate examples. Thereafter, the relation of Mathematics with daily life experiences has been discussed citing examples. As teacher, you may organise learning experiences by connecting with daily experiences of children in and out of the school context. At the end of the Unit, the correlation of Mathematics with other subjects has been discussed which would enable you to plan leaning activities integrating concepts belonging to other subjects. Integrated teaching facilitates children to develop mathematical concepts in a much better way.

2.10 UNIT END EXERCISES

1) Organise a discussion in your class on the topic 'enhancement of reasoning power of children'. Prepare a report.
2) Suggest a few strategies that can be organised in the classroom to promote the problem solving skills of children.
3) Ask children of your class to identify the mathematical ideas involved in their day to day activities
4) Identify a mathematical concept and explore the possibilities of teaching the same by integrating with other subjects.

2.11 ANSWERS TO CHECK YOUR PROGRESS

1) Refer section 2.3.1.
2) Following are the general objectives of teaching mathematics
   - Develop inertest and positive attitude towards Mathematics.
   - Apply mathematical knowledge to solve real life problems.
   - Develop the skill to use algorithms in problems solving.
3) The students will be able to:
   - Recall the definition of probability.
   - State the formula for probility.
   - Cite examples from daily life involving probability.
   - Solve problem related to probability.
4) Mathematisation refers to the act of interpreting or expressing mathematically, or the state of being considered or explained mathematically.
5) Role play may be organised. Children may be assigned different roles such as sales man, cashier, etc. Let them be involved in activities that occur in a textile showroom.
6) Refer section 2.4
7) Selection and formation of problem, Presentation of the problem, Formulation of hypothesis, Collection of relevant data, Analysis and organization of data, Formulation of a tentative solution, Drawing conclusion
8) Do it yourself.

9) Provide challenging mathematical problems; ask them to complete projects, etc.

10) Refer section 2.8

11) Refer section 2.8

2.12 REFERENCES AND SUGGESTED READINGS

- Anice, J.(2005). Teaching of Mathematics. Hyderabad: Neelkamal Publication Pvt.Ltd.
- Allendoerter, C. B., and Oakley, C. O.(1963). *Principles of Mathematics*. Tokyo: McGraw-Hill.
- Beckmann, C. E., Thompson, D. R. and Rubenstein, R. N. (2010).*Teaching and Learning High school Mathematics*. New Jersey: John Wiley and Sons Inc.
- Bloom, B.S.(1956). Taxonomy of Educational Objectives; The Classification of Educational Goals. New York: Longmans Green.
- DEP-SSA.(2009). *Teaching of Mathematics at upper primary level (Vol I and II).*New Delhi: Distance Education Programme-Sarva Shiksha Abhiyan.
- Haylock, D. (2010). *Mathematics Explained for Primary Teachers (fourth edition).*New Delhi: Sage Publications India Pvt.Ltd.
- IGNOU (2012).BES-009 *Teaching of Mathematics for the Primary School Child*, Block 1-4,SLM.New Delhi: IGNOU.
- NCERT (2005). *National Curriculum Framework-2005*. New Delhi: NCERT
- NCERT (2006). *Position Paper: National Focus Group on Teaching of Mathematics.* New Delhi: NCERT.
- NCTE (2009). *National Curriculum Framework for Teacher Education.* New Delhi: National Council for Teacher Education.
- NCERT (2012). Pedagogy of Mathematics, Textbook for Two Year B.Ed Course, New Delhi: NCERT.
- Hollands, R. (1965). Development of Mathematical Skills. London: Blackwell Publishers.
- Sidhu, K.S. (1967). The Teaching of Mathematics. New Delhi: Sterling Publishers.
- Mishra, L. (2008). Teaching of Mathematics. New Delhi: A.P.H Publishing Corporation.
- http://study.com/academy/lesson/critical-thinking-math-problems-examples-and activities.html
- http://assets.cengage.com/pdf/prs_clark-developing-critical-thinking.pdf
- http://edtechreview.in/trends-insights/insights/771-great-ways-to-teach-skills-like-critical-thinking-and-problem-solving
- http://tc2.ca/uploads/PDFs/TIpsForTeachers/CT_elementary_math.pdf

UNIT 3 HOW CHILDREN LEARN MATHEMATICS

Structure

3.1 INTRODUCTION

Who is an effective teacher? Can you say that a person who has an expertise in her/his subject is an effective teacher? Or one who is capable of helping the students to improve their learning?

Even a teacher with high academic profile may not be able to communicate effectively with students in a way that they can comprehend what she/he making them to learn?

What more is required for becoming a true teacher is that she/he is capable of knowing the students. Knowing students does not mean only acquiring information like students' names and ages, something about their friendship, family backgrounds, their academic record, etc.; but more than that, a teacher needs to have a deeper understanding of how the students learn in different situations, She/he must be well aware of what type of activities may be suitable for his/her students, what is the uniqueness in each child, etc. In other words, more than knowing about a child's personal information, understanding about his/her characteristic patterns of learning is important for a teacher.

In this unit, we will throw light on how important it is for the teacher to know children as learners. After going through this unit you will get enough opportunities to know how children learn, childrens' developmental sequence in learning mathematical ideas, and the processes involved in learning mathematical concepts.

3.2 OBJECTIVES

After studying this unit, you will be able to

- describe the ways children conceptualise various mathematical ideas;
- identify various situations that children learn Mathematics in a better way through experience;
- analyse that a child may utilise various strategies for learning;
- appreciate that a teacher needs to know the level of development of his/her learners;and
- demonstrate an understanding of processes involved in learning mathematical concepts.

3.3 CHILDREN'S CONCEPTUALISATION OF MATHEMATICAL IDEAS

You must have interacted with children in various contexts, inside as well as outside of the school. From your experience, do you believe that child's mind is a 'blank slate' when s/he enters a formal school? Children begin to learn Mathematics much before their formal schooling. Learning Mathematics depends heavily on the knowledge and experience that children bring with them to school. Children learn through numerous ways and as a teacher you are expected to have an understanding of this as well as of the pedagogical skills to use these experiences in your day-to-day classroom interactions. Let us discuss the ways used by children to internalise mathematical concepts.

3.3.1 Children Learn from Experiences

It is universally accepted that Mathematics is an abstract subject. The task of a teacher is to help his/her students conceptualise the abstractness through concrete experiences. If we are not in a position to provide ample opportunities to our students to concretise various concepts in Mathematics, they may condemn the subject as boring and difficult.

Let us see how two teachers taught the concept of the n^{th} term of an AP:

Teacher A

Mr Tomar is teaching Mathematics at secondary school level. He started taking the class with checking of home work, which usually takes hardly 2-3 minutes. He usually checks the work of students who completed it and gives some punishments to those who did not complete it. After this, he starts the new topic by saying "today we are going to learn how to find out the n^{th} term of an AP".

He wrote the formula for calculating the n^{th} term of an AP $T_n = a+(n-1)d$ where 'a' is the first term, 'd' is the common difference, and 'n', the n^{th} term. Then, he asked his students to copy it. Then it is followed by a problem which was solved *by the teacher through discussion with students and subsequently solution was copied by the students.*

Then Mr. Tomar gives a problem to the whole class as a class work. Students independently start solving it, and the teacher is not at all involved in the process. This activity ends as soon as two or three students solved it correctly. Mr. Tomar ends the class after providing them with homework from the text book.

Teacher B

Mr Jitendra, another teacher who is also teaching Mathematics at secondary school level has evolved a different pedagogical process while teaching the same concept of n^{th} term of an AP to his students.

He started his teaching with discussion on the homework of students one by one. He asked students to explain how they did the home work and what were the answers etc, and also asked the students who had not done home work and asked to complete it during free time or at home.

Then, he started the class with discussion with the students about what they learned in the earlier class. What is an AP? How an AP can be constructed? He asked one student to write the first four terms of an AP, and another student to write the next four terms etc. This is followed by asking the class to find out its 100^{th} term.

Some of the students started to find the 100^{th} term, then he told the class that it may take much time to find out the 100^{th} term if you follow this way. Can you find out another easy way to calculate it?

He facilitated the discussions through appropriately guiding the students to derive the formula for finding the n^{th} term.

The discussion went on like this,

Second term is nothing but the first term +common difference (cd)

Third term is second term+ cd, which is equivalent to first term + 2times cd

Fourth term is equal to third term+cd, which is equivalent to first term + 2times cd+ cd, or first term+ 3 times cd

a, a+d, a+2d, a+3d .... a+(n-1)d

1^{st}, 2^{nd}, 3^{rd}, 4^{th} .... n^{th}

……………………………………………………………………………………..

……………………………………………………………………………………..

……………………………………………………………………………………..

……………………………………………………………………………………..

……………………………………………………………………………………..

This activity is followed by solving two or three problems by the students as a class work. The rest of the problems were given as home work.

A close examination of the pedagogical processes used by the two teachers reveals that, the teacher A has never considered learner as an active partner in the teaching-learning process and ignored the importance of students own experiences in conceptualising mathematical knowledge. Teacher B, on the other hand has given more and more opportunities to the students to construct knowledge and improve their conceptual understanding through discussions and peer interactions. So without any doubt one can say that the second teacher's approach is better.

Let us see another example, a teacher wants to teach the concept that sum of the angles in a quadrilateral is 360º.

What pedagogical process can teacher use so that students can conceptualise the idea?

One way is that to draw a Quadrilateral on BB and can measure its angles using protractor and show that sum of the angles is equal to 360º

Another way is teacher can draw a quadrilateral on a piece of paper or hard board and draw a diagonal so that the quadrilateral can be divided in to two triangles. Then use angle sum property of triangles to show that sum of the angles of a quadrilateral is 360º.

Third way is teacher can ask students to construct various types of quadrilaterals on a piece of paper or chart. Students can be asked to measure the angles and find out the sum of the angles in the case of each type of quadrilaterals. From the various examples they may be in a position to generalise that sum of the angles is equal to 360º. They can prepare following chart based on their experience

Name of the Quadrilateral	***Measure of Angle 1***	***Measure of Angle 2***	***Measure of Angle 3***	***Measure of Angle 4***	***Sum of the angles***
Rhombus					
Square					
Parallelogram					
Rectangle					
Trapezium					
Kite					
Quadrilateral					

Out of these three experiences, which one will be more helpful for students to learn the concept better and more effectively? The role of students during the first two cases was mere passive listeners whereas in the last case students were actively involved in the process, and learning takes place through their own experiences.

As a teacher it is our duty to provide such type of activities give first hand experience to the learner to construct mathematical knowledge. What is required here is that we need to provide opportunities to the students of actively participating in the learning process. If they feel that 'they themselves' derived the formula, the satisfaction that they would get will definitely influence their approach to learning Mathematics. No doubt, they will show interest and positive attitude towards the subject. **The teacher needs to provide the students with varied types of experience in a Mathematics classroom, like group discussion, problem-solving exercises, mathematical games, puzzles** etc.

The experience of success through these activities result in the following benefits:

- It enhances their mathematical ability.
- Students will be intrinsically motivated.
- Children will show interest in solving mathematical problems and reading literature.
- Children will actively participate in Mathematics club activities, exhibitions, quiz, etc.
- Children will show positive attitude towards Mathematics.
- It will boost the children's confidence in learning Mathematics.

Check Your Progress

Note: a) Answer the following question in the space given below.

b) Compare your answer with those given at the end of the Unit.

1) What are the benefits ,when children learn mathematics by using their own experiences?

....

....

....

....

....

....

....

3.3.2 Children have their own Strategies for Learning

Renu, a class 9 teacher, after teaching algebraic identities gave the following problem to the students:

'Evaluate 84 x 85, without multiplying directly'.

Visnu did it in the following way;

$$84 \times 85 = (80 + 4)(80 + 5) = (80)^2 + (4+5)\,80 + 4\times5 = 6400 + 720 + 20 = 7140$$

Sonu solved it in this way

$$84 \times 85 = (90 - 6)(90 - 5) = (90)^2 - (6 + 5)\,90 + 6 \times 5 = 8100 - 990 + 30 = 7140$$

Monika used another method for solving it:

$$84 \times 85 = (80 + 4)(90 - 5) = (80 \times 90) - (80 \times 5) + (4 \times 90) - (4 \times 5) = 7200 - 400 + 360 - 20 = 7560 - 420 = 7140$$

What do you see in these three approaches? Whose method is correct? and why?.

As a constructivist teacher we have to agree that all methods are correct and the method followed by Monika is more creative and innovative since the other two methods might have been explained by the teacher in the classroom.

If you give freedom to the students to work independently and encourage and help them continuously in the learning process, surprisingly, you can see that students will come up with solutions for a particular task or problem in different ways.

You may be aware of the story of Gauss, the famous mathematician. When he was a primary school student, his teacher asked the class to find out the sum of first 100 natural numbers. The teacher had an important work to complete, so he decided to give that problem so that students would take more time for completing it. But, surprisingly Gauss solved the problem quickly, and when the teacher asked about the solution, he explained that he added the pair of numbers from beginning and end and formed 50 such pairs, the sum of each being is 101 (1+100, 2+99, 3+98,…….50+51). Hence, total sum is 101x50 = 5050. Teacher doesn't waste time in congratulating Gauss in front of all students. The support and facilitation received from teachers and others during the formative years of his life energised Gauss to come up with more and more creativity and innovative ideas in Mathematics subsequently.

Instead of that, suppose **we are discouraging our students against using their own innovative strategies in Mathematics classroom, definitely that will result in blocking their ability to think divergently**, **to try for alternatives and arrive at solutions in an innovative way.** That automatically result in forcing students to memorise formulae, definitions, procedures, etc through rote memorisation without understanding the ideas meaningfully. This will not help the students to use those mathematical concepts for a longer time, instead, it may help them to score marks in the immediate examination. As teachers, we should be clear that our purpose or duty is not to help the students to score high marks in examination in a routine way, but help them to comprehend the ideas in a meaningful way, so that our students may succeed in their life. For attaining this aim, **we need to consider the ability of our students, and as far as possible, various activities should be provided in the classroom in order to enhance the creative abilities of the students.**

While doing so in our classrooms, we should not expect that our students will come up with correct solutions. In fact, sometimes they may be wrong also, but they may not be aware of it. What can we do in this situation? Should we blame our students for solving in their own way instead of following the way we discussed? Of course, no, what we should do is **to encourage our students to seek alternative strategy and facilitate them to arrive at solutions in their own way.** If we do so, the students will definitely try to modify old strategy and develop a new one to arrive at correct solution.

Check Your Progress

Note: a) Answer the following questions in the space given below.

b) Compare your answers with those given at the end of the Unit.

2) 'Children learn from experience'. Substantiate this statement by taking the example of learning any concept in statistics.

....

....

....

....

3) Select any problem from trigonometry and show that children can solve it in a different way.

....

....

....

....

....

....

3.3.3 Children See Mathematics Around Them

One day, while in an informal discussion, one of my friends, a faculty member in a B. Ed. college, illustrated her experience. She was teaching the topic 'Measurement and Evaluation', in which the statistical topics like measures of central tendency, measure of dispersion etc. were also involved. She told me that it was considered as difficult and very boring by most of the non-mathematical students. She tried her best possible way to explain the concept of 'Mean' using various examples from classroom such as marks of the students, weights of the students etc. Most of the students understood the concept. A few students said that they were not in a position to comprehend it. Dinkar, a history graduate, whom she knew previously also, was one among them. Dinkar had a business of poultry, which she knew. She asked Dinkar about his business and he responded that he was dealing with chicken as well as eggs. She asked him the number of eggs he was able to sell daily on an average. After a few seconds of silence, and some mental calculation, he told her that it was around 40. When she asked him about how he reached the answer he told that every day he was selling 35 to 45 eggs, and hence the answer. She utilised this example to make the students conceptualise the idea of 'mean' further.

The above incident is not a single one; we can see lot of such examples in our classrooms. If we want to conceptualise an idea, it is better to connect the students with their immediate environment.

Students are coming to the classroom with a lot of exposure to the environment. In each and every moment of our life, knowingly or unknowingly we use Mathematics. Whether it is at home, play ground, office, railway station, bus journey, or in a party, we are bound to use different types of Mathematics. Still, most of our students are showing an aversion towards Mathematics. Hence, being Mathematics teachers, we should carry forward whatever Mathematics students know from their own environment by connecting it with the topic we are teaching. **The teacher who connects mathematical concepts with the objects around the students, can be considered as a successful teacher.**

You can give numerous instances in which we use Mathematics in our daily life. **What more is required is judiciously utilise the various experiences of students in constructing mathematical concepts.** The difficulty of learning Mathematics and negative attitude towards the subject will slowly decline if we are able to connect mathematical concepts with that of students' daily experiences.

3.3.4 Every Child is Unique

Have you heard about the concept of 'Tabula rasa'? It means a clean slate, ie nothing written on it. Our students are not coming to the class like a Tabula rasa. Even an infant's mind is not empty. Each and every moment in their life children are experiencing many things, and as a result of that they are learning. In the earlier sub-section, we discussed that child learns Mathematics from his daily life activities. Every child might be getting various kinds of exposure, and as a result of that, she/he may be conceptualising different things also. Thinking process of one child may not be the same as that of the other. One child may be learning mathematical concepts while playing cricket with their peers. Yet another may not be seeing Mathematics in a cricket game, but may be applying mathematical concepts while drawing pictures. Since our children are unique in their personality, necessarily they may have uniqueness in their thinking also.

You might have experienced this in your own classroom. **You cannot find out two children with the same characteristics in all the aspects.** You may be able to find two children with same weight, height, date of birth, and scoring the same marks in tests. But if you observe them critically and analyse their skills in applying various mathematical processes, you will find that, in spite of all these similarities they show differences. One may solve numerical problems quickly, but may not be able to perform at the same pace, when asked to solve a word problem.

Activity for Practice

Observe any two students of the same ability level in Mathematics from your class for at least one week, and see how they differ in their approach in various aspects of Mathematics learning.

Check Your Progress

Note: a) Answer the following questions in the space given below.

b) Compare your answers with those given at the end of the Unit.

4) Prepare your daily schedule of activities and enlist at least 10 mathematical concepts that you can relate to it.

....

....

....

....

....

5) From the secondary Mathematics syllabus, list at least five concepts that are related with our real life experiences.

....

....

....

....

....

3.4 DEVELOPMENTAL PROGRESSION IN THE LEARNING OF MATHEMATICAL CONCEPTS

In the first section we discussed how the children conceptualize mathematical ideas and the importance of conceptual knowledge in Mathematics learning. Let us go back to section 3.3.1; there we illustrated how Mr. Tomar and Mr.Jitendra taught their students about the concept of the n^{th} term of an AP. The pedagogical process used by Mr. Tomar was completely teacher oriented without any scope for students to reflect or question anything. Students were passive listeners and learning become more and more difficult for them. This pedagogical approach is a contribution of behaviourist psychologists. On the other hand the pedagogy used by Mr. Jitendra was completely student oriented and it was due to the active participation of students, they derived the formula of the n^{th} term of an AP. The concept they developed in this way will be ever lasting. The psychological approach used by Mr. Jitendra is 'Constructivism'. In fact as Mathematics teachers we also need to conceptualize constructivism and the way it proposes the progressive development of concepts.

Constructivism, as a theory of learning, believes that each and every individual has the capacity to construct his own understanding and knowledge about various things through experiencing and reflecting. In this process, whenever a child encounters a new experience, he/she can either easily connect it with the existing knowledge or can make some changes in the existing knowledge to accommodate the new experience.

In the following subsection, we can discuss the contribution of three famous constructivist psychologists, viz, Piaget, Bruner, and Vygotsky to the process of cognitive development.

3.4.1 Jean Piaget's Views

According to Piaget, cognitive development is synonymous with changes in cognitive structure, where a **cognitive structure consists of several isolated structures, called schemes, that undergo both quantitative and qualitative changes during development. Schemas are the basic building blocks of thought and the mental representation of the objects and events in the world** and may be discarded or modified or retained as a result of experiences.

For example, 'number' is a schema, in our cognitive structure from the first standard onwards. How is this schema undergoing modification as one reaches up to graduation level. For a first standard student, it represents any of the natural numbers, when he or she reaches fifth standard, a number may be natural number, may be integers, may be whole number etc.

According to Piaget, one's cognitive structure consists of different kinds of schemas, and cognitive development occurs as a result of changes in the cognitive structure as well as due to experiences gained during interaction with the physical world. What are the processes involved in this modification process?

Assimilation and accommodation are two technical terms used by Piaget to explain how cognitive structure undergoes modification and improvement. **Assimilation occurs when people try to understand a new experience by**

matching it with existing schemas. Accommodation takes place when a person brings about changes in these existing schemas to adapt to a new situation.

For example a student has the concept of rational number. When he/she learns the concept of percentage, he/she will be able to assimilate it into the existing schema. He/she can easily internalise the concept of percentage, by linking it with rational number. Even though both represent different schemas in the cognitive structure, it can be stored in such a way that a percentage is a particular rational number whose denominator is always 100? Through this process, he/she will be able to accommodate the new schema.

Suppose a student assimilates a new thing, which is contradicting or confusing with some shemas already there in his/ her cognitive structure.Then the child will experience a conflicting situation, which Piaget called as *cognitive conflict*. This will leads to a condition of *disequilibrium*. As a result of this child will try to find a suitable place for the new schema through a process of reorganisation and restructuring of the previously existing schemas. This process of finding a new place for the assimilated schema is known as accommodation. Through this way, the cognitive structure will become in the equilibrium position again.

In the above example, after introducing the concept of percentage, students may have some confusion since they represent 2% as 2/100, whereas, they learned as a rational number. This confusion will be cleared when they understand that percentage is a special type of rational number.

Thus, the **cognitive structures are constructed and continuously reconstructed through an interaction between the student and various experiences in and out of classroom.** As a result of modification in the cognitive structure, more and more concepts will be assimilated and accommodated. Based on his studies, Piaget put forth clearly demarcated sequential stages in cognitive development, namely:

- ✓ Sensory motor period (Birth to 2 years)
- ✓ Pre-operational period (2-7 years)
- ✓ Concrete operatioal period (7-11 years)
- ✓ Formal operational period (11-15 years)

Sensory Motor Period (Birth to 2 years) : Children in sensory motor period learn mostly through trial and error learning. Children initially rely on reflexes. In the later part of this period the child will be able to mentally represent the objects and events, and the process is called *object permanence*. For example, if you hide a toy at any place in front of the child, the child who has attained the object permanence stage knows the place where it is kept.

Pre-operational Period (2-7 years) : Children in the pre-operational stage can mentally represent events and objects and engage in symbolic play. The typical characteristics of pre- operational children are:

- Able to use symbols (language),
- Engaged in symbolic play .
- Egocentric (view of world from one point of view)

- Thinking unsystematic / illogical,
- Centers on one aspect of object / problem at a time,
- Cannot reverse processes, but has rediments of conservation and classification,
- Does not have a structure of whole, but rather many isolated segments.

Concrete Operatioal Period (7-11 years): During this stage the child

- Can conserve mass, length, weight and volume,
- Able to reverse and decentre,
- Can classify objects (organise objects into an organised schema),
- Logical thinking based on direct experiences.

Formal Operational Period (11-15 years): During this stage child exhibits

- Hypothetical / deductive reasoning (can identify possible solutions to problem solving, can test systematically),
- Inductive reasoning (can move from specific facts to formulate general principles and conclusions),
- Reflective abstractions (can reflect on self/what might happen),
- ability to reason in purely symbolic / abstract manner.

According to Piaget, Mathematics, and infact, most of the essential schemas cannot be 'taught', they have to be 'constructed' by the child. In the early stages verbal instruction may not help much. Various types of activities, which are essential for building shemas should be included. In understanding a problem the child assimilates it into his/her existing schemas and incorporates into his total cognitive world. When the existing schemas are inadequate to the complexity of the problem, 'mistakes' occur. Then non-piagetian teacher will concentrate on occuring the mistake. The Piagetian teacher will help to create the condition under which new schema will be created which can deal with the new stimuli. The child studying under a traditional clasroom may seen to learn certain things faster by mechanical means. But, **the learning of the Piagetian child will be firm and generative and hence in the long run the Piagetian child is likely to overtake the child learning by traditional methods**.

3.4.2 Lev Semionovich Vygotsky's Views

Vygotsky argued that **as a result of the social interaction between the growing child and other members of the society that the child acquires the tools of thinking and learning**. His theory is primarily based on the concept of 'Zone of Proximal Development'.

Let us consider the following example:

Teacher gives the following problem to the 10^{th} class students as a class work.

'A kite is flying at a height of 40m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60º. Find the length of the string, assuming that there is no slack in the string'.

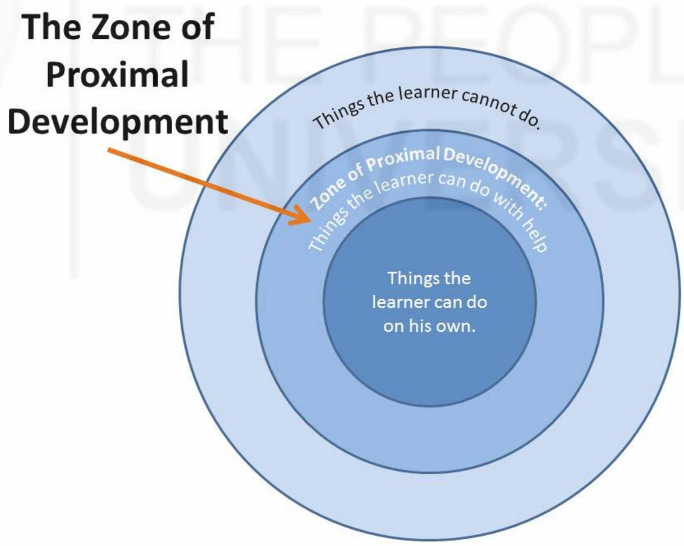
One can assume the following possibility as far as the problem solving activity is concerned.

- All students can solve and complete it individually without any external support.
- Some students can solve it individually, but some need external support to solve it.
- All need external support in order to solve the problem.

As teachers, we have to expect these three groups in our classrooms depending on the situation or problems.

Let us consider the first case; here all students can solve it individually without taking any help from outside. Suppose the problem is very simple or teacher has provided a very simple activity to perform, all students from the class may be accomplishing it individually. According to Vygotsky, every learner at one stage **has the capacity or ability to construct knowledge by himself/ herself when confronted with an experience without any other external support**. He called this level of knowledge construction as *'Level of Actual Development'*. We can say that students of the first case are in their Level of actual development. If the difficulty level of the problem or task increases, we can see that some of our students may not be in a position to solve it independently, but if we guide them in a proper way, they may be in a position to solve it. This indicates that a **student can even reach beyond his or her Level of Actual Development, with the help of some support from teacher or any knowledgeable person.** The level up to which a student can reach in this way, is called *'Level of Potential Development'*.

One can see a zone between these two levels which Vygotsky called *'Zone of Proximal Development (ZPD)'*. **ZPD is nothing but the difference between the Level of Potential Development and the Level of Actual Development.**



As a teacher, how you will use the concept of ZPD while planning classroom process?

Vygotsky was of the opinion that development is more dynamic and effective if children are exposed to new learning, specifically, in their proximal zone of development. In this zone, with assistance from the teacher or other knowledgeable persons, and peers; children would be able to assimilate more easily what they would be incapable of assimilating if left to themselves. The assistance may in the form of demonstration, giving more examples, monitoring and providing feedback, shared activities, etc.

3.4.3 Jerome S Bruner's Views

According to Bruner, society and its culture play an important role in the development of cognition in a child. Like Piaget, Bruner also formulated sequential developmental stages based on the stage of representation. **He argued that cognitive development moves through three stages: *inactive, iconic,* and *symbolic.* However, unlike Piaget's stages, Bruner argued that these stages were not necessarily age based.**

First stage is known as the stage of inactive representation; here the child knows the world by the habitual action he or she uses for dealing with it. In this instance he 'knows through doing things.' That means **knowledge is developed and stored basically in the form of motor responses.** Second stage is known as 'iconic' representation. In this stage, the child begins to represent the world through images as spatial schemes. The knowledge is basically stored through visual images. In this stage students will be able to learn quickly if the teacher can assist them to conceptualise it with the help of pictures, diagrams, graphs etc. The third stage is known as 'symbolic' representation; during this stage the child can translate actions and images into language. Here **knowledge is stored in the form of words, mathematical symbols etc.**

Bruner put forward the idea of the spiral curriculum, in which modern concepts may be presented even to young children, but revisited at higher stage in greater depth and breadth. He was not happy with the Piagetian view that educators should wait for the child to be ready to learn. Instead he proposed a much more active policy of intervention through spiral curriculum. He holds that it is possible for the ordinary teacher to teach the ordinary child, in the ordinary school the structure of subject, which will help the child to generate much of the content, instead of memorizing too much unrelated facts. He was of the opinion that 'any subject can be taught effectively in some intellectually honest form to any child at any stage of development'.

Check Your progress

Note: a) Answer the following questions in the space given below.

b) Compare your answers with those given at the end of the Unit.

6) Explain the concepts of assimilation and accommodation with the help of examples of polynomials.

....

....

....

....

....

....

....

....

7) You have given a problem to the whole class to do. During observation, you noticed that most of the children are doing it correctly. But a few asked for help from you. Is the problem enough for understanding the ZPD of all students? If yes, how? If no, why?

....

....

....

....

....

....

....

8) According to Bruners, What are the different stages of cognitive development?

....

....

....

....

....

....

....

3.5 PROCESS INVOLVED IN LEARNING MATHEMATICS

The realisation of the aim 'mathematisation' will by and large depends on how our children learn Mathematics. Mathematics being an abstract subject requires systematic and logical approaches from the teacher in order to help the child to crack the abstractness with ease and confidence. Mathematics is not merely a collection of content; instead it is more of processes that are essential for comprehending mathematical concepts. We will be discussing more about these processes in this section.

3.5.1 Problem Solving

Problem solving is considered as a process in order higher level thinking and reasoning. It comprises sequence of steps: Understand, Think, Try and Look Back. Problem solving is not a skill that one is born with – it is cultivated over time with practice and experience. However teachers can help children by providing space and opportunity to do tasks which require problem solving. We have already discussed different steps of problem solving with examples in Section 2.6 of Unit 2.You may refer and oriagnise teaching learning activities for developing problem solving skill among your students.

3.5.2 Patterning

Most of the basic mathematical concepts can be introduced with the help of patterns. Students show interest and active involvement in an activity which requires to devise pattern and extend it. Devising a pattern is very crucial in Mathematics for 65eneralization.

You might have heard of series 1, 1, 2, 3, 5, 8, ……………, which is popularly known as Fibonacci series. The beauty of this series can be seen in our nature. For example, In the case of pineapples, we see a double set of spirals – one going in a clockwise direction and the other in the opposite direction. When these spirals are counted, the two sets are found to be adjacent Fibonacci numbers.

Fig. 3.1: Picture of a pineapple

Students should be provided with opportunities to identify and relate patterns in whatever they may be doing.

For example consider the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Can we help the students to expand $(a + b)^8$?

With the help of patterns in the previous cases, the students can be encouraged to generalise the pattern to expand $(a + b)^8$

Let us start with $(a + b)^0 = 1$

Write the coefficients of all the terms in the expansion in a row

1

Consider $(a + b)^1 = a + b$

Write the coefficients of all the terms in the above expansions in a row as given below:

We get

1

1 1

Now, consider the identity $(a + b)^2 = a^2 + 2ab + b^2$, and write the coefficients of its terms in the expansion just below that it.

1

1 1

1 2 1

Next write the coefficients of terms in the expansion of $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ just below it.

We get, a pattern like this:

1

1 1

1 2 1

1 3 3 1

Students can easily complete the next row of this pattern, which will consist of the coefficients of terms in the expansion of $(a + b)^4$.

At the same time, students can observe a pattern in the degree of each term and 'a' and ' b' separately.

For $(a + b)^0 = 1$, the degree of 'a' and 'b' is 0

For $(a + b)^1 = a + b$, the degree of 'a' and 'b' is 1

For $(a + b)^2 = a^2 + 2ab + b^2$, the degree of 'a' is 2, in the first term and is decreasing to1 in the next term and to 0 in the third term. At the same time, the degree of 'b' is 0 in the first term, increased to1 in the second term and to 2 in the third term.

In the case of $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, degree of 'a' is 3 in the first term and decreases by 1 each in the subsequent terms with a degree of 0 in the last term. At the same time the degree of 'b' is 0 in the first term, increased by 1 in each of subsequent terms, and is 3 in the last term.

Students can easily recognise patterns in degrees as well as coefficients, and can expand any algebraic expression.

In fact, for expanding $(a + b)^8$, they have to complete the pattern up to the ninth row first

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 10 21 7 1

1 8 28 56 70 56 28 8 1

Through creating patterns, students can find out the coefficient of various terms in the expansion. They already found that the degree of 'a' in each term will decrease by one and that of 'b' will increase by 1 as shown in the pattern.

Hence the expanded form will be

$(a + b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$

3.5.3 Reasoning

Mathematical thinking is characterised by reasoning. **Reasoning can be considered as a process of drawing conclusions on the basis of evidence.** Most of the proofs and generalisations in Mathematics require reasoning. Mathematical reasoning is of two types, inductive reasoning and deductive reasoning. Mathematics being a subject abstract in nature requires the help of inductive reasoning to arrive at various abstract concepts. **Inductive reasoning starts with a specific example or case and leads to generalisation. It consists of four stages, viz, Presentation of specific examples, observation of their characteristics and figuring out the common characteristics, generalisation, and verification.**

Ms Sunita, was teaching the concept of angle-sum property of triangles. She decided to organise a group activity for helping all students to internalise the concept. For that purpose students were divided into five groups. Each group was given a set of 6 triangles. They were asked to measure the three angles of each triangle using the protractor and write the results in the space provided in a table. After measuring each angle they were asked to complete the given table as given below:

Triangle	**Angle 1**	**Angle 2**	**Angle 3**	**Total**
ΔABC				
ΔDEF				
ΔJKL				
ΔMNO				
ΔPQR				
ΔXYZ				

After completing the activity in the group, students were asked to discuss within the group what they found and what would be the sum of the angles, if teacher gave them another triangle.

Thus, by considering specific cases, students found that sum of the angles of any triangle is 180^o.

Deductive reasoning is the opposite of inductive reasoning process. In this process, we start with a general principle, formula or statement, and draw valid conclusions about specific examples. This process helps the learner to concretise the abstract concept which they learned.

For example, students learned the algebraic identity

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

From this they can expand $(3x + 2y)^3$ in the following way

Here a = 3x and b = 2y

Therefore, $(3x + 2y)^3 = (3x)^3 + 3(3x)^2(2y) + 3(3x)(2y)^2 + (2y)^3$

$= 27x^3 + 54x^2y + 36xy^2 + 8y^3$

Caution: We need to understand that there is no guarantee that all of our inductive reasoning will give us correct generalisation.

3.5.4 Abstraction

Mathematics is considered as an abstract subject. Most of the concepts in Mathematics are abstract in nature. How can one reach this abstraction and learn higher and higher related concepts? Unless the students learn the basic concepts in Mathematics, learning the complex concepts will not be possible.

Let us see how a primary school student learns the concept of 1. For a novice child, the concept will be abstract in nature. Teacher, through various concrete examples, makes the child conceptualise the idea of oneness. He or she may be shown one pencil, one flower, one ball, etc and through such type of examples gradually develops the concept if one. Later on, he or she will be in a position to use this abstract concept in developing other abstract concepts like 2, 3, 4, etc, and in later classes, various other concepts of numbers and variables in algebra.

Hence, we can say that, **specific or concrete cases help us in the process of abstraction.** In other words, inductive reasoning in Mathematics ends with abstraction. From the abstract concepts, we can develop more and more related abstract concepts.

For example, quadrilateral is an abstract idea and we concretise it with the help of various examples of four sided and other closed figures. After the formation of the concept of a quadrilateral, the same can be utilised for developing the concepts of square, rectangles, trapezium etc through establishing relationships.

3.5.5 Generalization

In the earlier sub sections, we discussed the process of inductive reasoning, patterning and abstraction. Do you see any commonalities in these mathematical processes?

A closer examination of these processes will lead you to a conclusion that in all these processes the result or the outcome is generated on the basis of generalisation from specific cases, examples or situations. **Generalisation is the process of identifying a pattern or a relationship and extends this pattern or relationship beyond the given cases.** In the case of inductive reasoning and abstraction, the generalisation emerges out of particular examples or cases. In patterning, each stage of the pattern gives us a clue about the next stage and so on.

You may be remembering that we have already discussed in section 1.4.2 of Unit 1 about the concept of generalisation and how mathematical generalisation helps in the process of reasoning.

3.5.6 Argumentation and Justification

Abstract nature of Mathematics has been discussed in an earlier sub section, and in Mathematics one come across generalisation of mathematical concepts at different occasions. Argumentation and justification can be considered as two important components of a mathematical proof and generalisation.

A constructivist classroom will provide ample opportunities to the students for discussion and communication with peers and teacher. During these discussions, students or a group, after an activity can share the procedure followed by them, the strategy used, their observation, and others can agree or disagree with their view points. The other group can argue with the group as to why they disagree with proper justification. At the same time, the group which presented also can argue with logical reason. The group convinces others that their arguments are correct and valid with the help of concrete evidence.

The processes of argumentation and justification not only help the students to internalise proof of a theorem, principle or formation of concepts, it also helps to develop a habit of reasoning, while dealing with various real life situations.

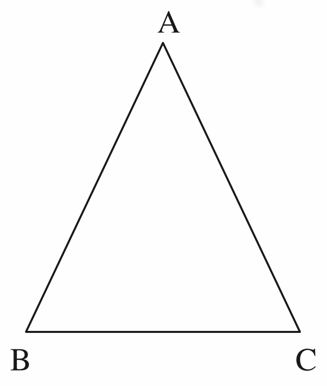
Justification is an essential component in problem solving as well. Why I am using a particular strategy for solving a problem needs to be justified with valid reason. Again whether the solution obtained is justifiable or not is also important. Many times our students will not justify the answer. Common practice is that, students will justify the answer only when the teachers ask them to do so due to mistake or error. What is required is to build the process of justification as a habitual one in our students. The process of justification will help them to explain why their answer is correct and convince the peers and teacher.

The example given below shows, how argumentation and justification are used by a child while proving angle sum property of triangles.

Theorem : Prove that sum of the interior angles of a triangle is 180°.

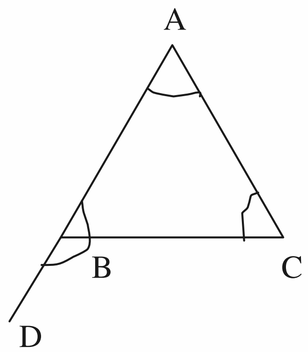
Proof

Consider the triangle ABC.



We have to prove that A+ B+ C = 180°

Extend AB to D



From the above figure ∠CBD is an exterior angle. (By the definition of exterior angle).

We know that measure of the exterior angle = sum of the measures of the two opposite interior angles

Therefore ∠A+∠C = ∠CBD (∠A and ∠C are the two opposite interior angles)

Hence ∠A+ ∠B+ ∠C = ∠CBD+ ∠B (By adding m ∠B on both sides)

But ∠CBD+ ∠B = 180° (∠CBD and ∠B are linear pairs)

Therefore, ∠A+ ∠B+ ∠C =180° (RHS = LHS)

Hence, the sum of the three angles of a triangle is equal to 180°.

In the above proof, we can see that the argument placed by the student in each step has been substantiated with proper justification.

Art of learning Mathematics depends on how the child has been provided opportunity to experience these processes. Mechanical learning will not help the child to conceptualise the meaning of Mathematics content. The responsibility of the teacher rest on, facilitating the student learning through organising different as well as challenging learning tasks to students.

Check your progress

Note: a) Answer the following questions in the space given below.

b) Compare your answers with those given at the end of the Unit.

9) Patterning can be used as an effective strategy for generalisation of mathematical ideas. Can you substantiate this statement with a suitable example from secondary school Mathematics?

…………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

10) Differentiate between Inductive Reasoning and Deductive Reasoning. In your opinion, which process is better for learning Mathematics?

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

3.6 LET US SUM UP

In this unit, an attempt was made to familiarise you with various ways through which a child learns various mathematical ideas. We discussed the importance of child's experiences in learning Mathematics and various strategies for solving a mathematical problem or task. An attempt was made to discuss the theories of Piaget, Bruner and Vygotsky with special attention to their contribution to child's cognitive development. In the last sub-section, some of the processes involved in Mathematics learning have been discussed. Some of the major points discussed in the unit are:

- Every child learns through his /her own experience and then sometimes device his/her own strategies for learning.
- Children come across situations in their every day affairs where they use some mathematical knowledge.
- Teachers need to consider the ability of their students, and as far as possible various activities should be organized in the classroom in order to enhance the creative abilities of students.
- The teacher, who connects mathematical concepts with the objects around the students, can be considered as a successful teacher.
- Cognitive structures are constructed and continuously reconstructed through interaction between the student and various experiences in and out of the classroom.
- ZPD is nothing but the difference between the Level of Potential Development and the Level of Actual Development.
- Bruner argued that cognitive development moves through three stages: *inactive, iconic,* and *symbolic.* However, unlike Piaget's stages, Bruner did not argue that these stages were necessarily age-based.
- Inductive reasoning starts with a specific example or case and leads to generalisation.

- In deductive reasoning we start with a general principle, formula or statement, and draw valid conclusions about specific examples.
- Generalisation is the process of identifying a pattern or a relationship and extend this pattern or relationship beyond the given cases.
- Argumentation and Justification can be considered as two important components of a mathematical proof and generalisation.

3.7 UNIT END EXERCISES

1) What are the importance of own experience in learning of mathematical concepts?
2) Explain the features of formal operational period with examples from Mathematics.
3) Elaborate the concept of 'ZPD'.
4) Discuss the role of Inductive reasoning in learning Mathematics.
5) Differentiate between assimilation and accommodation.

- 6) What is argumentation and justification? Explain with examples.

3.8 ANSWERS TO CHECK YOUR PROCESS

1) Children will show interest in solving mathematical problems and reading literature; Children will actively participate in Mathematics club activities, exhibitions, quiz, etc.; Children will show positive attitude towards Mathematics.

2) There are plenty of examples from child's life, which can be utilised for helping child to learn statistics. Child knows that his/her weight is different from that of the peers. Suppose a situation occurs in which a single number representing the weight of the students from their class require. The concept of Mean can be introduced with the help of this situation

3) Find the value of sin 405^o

a) $\sin 405^o = \sin(360+45^o) = \sin(4x90+45^o) = \sin 45^o = 1/\sqrt{2}$

b) $\sin 405^o = \sin(450 - 45^o) = \sin(5x90- 45^o) = \cos 45^o = 1/\sqrt{2}$

4)

Sl No	Activity	Time	Mathematical Concept
1	Wake up	**5.30 am**	**Time**
2	Brushing the teeth	5.40am	Profit, Loss, etc
3	Morning Tea	5.45 am	Ratio
4	Morning Walk (Jogging)	5.50am-6.20am	Number, Average, Speed etc

5) Linear Equations, Area and Volume, Circle, Statistics, Probability

6) Assimilation is the process of incorporating new information into the previously existing schema. Here, the student fits the new idea into what he/she already knows. For example, student learns the concept of polynomial. Accommodation is the process by which pre-existing knowledge is altered in order to fit in the new information. For example, the student already has the idea of simple equations. While learning polynomial he/she would connect it simple equations and differentiates both.

7) If most of the students are able to solve the problem by themselves, then the problem lacks the feature of understanding the ZPD. At the same time some students ask for help indicates that, the problem works in the ZPD of those students.

8) Inactive, Iconic and Symbolic

9) Students can be asked to see the effect of changing the radius of a circle on its perimeter by measuring the radius and perimeter of various circular objects. They may be asked to prepare a chart showing the radius, perimeter and their ratio. From the pattern let them generalise about the ratio

10) Inductive reasoning is an approach in which generalisations are based on examples. It starts with particular/example/concrete/known and arrives general principle. It is also known as 'bottom-up' logic. On the other hand Deductive reasoning starts from generalisation/ principles/ unknown/ abstract etc and derives the specific examples/ concrete /known. This is also known as 'top-down' approach.

In learning Mathematics both types of reasoning are vital. Inductive reasoning is important while introducing any concept especially during school stages. Deductive reasoning needs to be utilised while solving different types of problems.

3.9 REFERENCES AND SUGGESTED READINGS

- Anthony, G. and Walshaw, M, (2009). Characteristics of Effective Teaching of Mathematics: A view from the West, *Journal of Mathematics Education,* Vol 2, No.2, pp147-164
- Beckmann,C.E, Thompson,D.R and Rubenstein, R. N. (2010). *Teaching and Learning High School Mathematics,* John Wiley and Sons Inc. New Jersey
- Chambers Paul (2010). *Teaching Mathematics- Developing as a Reflective Secondary Teacher*, Sage South Asia Ed, New Delhi
- NCERT (2005). *National Curriculum Framework, 2005,* NCERT,New Delhi
- NCERT (2005). *Position Paper National Focus Group on Teaching of Mathematics*, NCERT, New Delhi

- NCERT (2012). *Pedagogy of Mathematics: textbook for two year B Ed Course,* NCERT, New Delhi
- Wilson, B. G. (1997). *Reflections on constructivism and instructional design, instructional development paradigms,* Educational Technology Publications. Available
- WWW: [http://carbon.cudenver.edu/~bwilson/construct.html].

UNIT 4 MATHEMATICS IN SCHOOL CURRICULUM

Structure

4.1 INTRODUCTION

In the earlier units, we have discussed the nature, aims and objectives of Mathematics. You all are aware that Mathematics is considered as the mother of all sciences. Why is it so considered? This should be explained to learners. Learners should be provided an opportunity to explore the aims and purposes of teaching-learning Mathematics.

Learners at secondary level should understand the importance of keeping Mathematics at a key position in school curriculum. They should explore the essential topics those to be included in the curriculum at various levels of school education. This Unit gives careful thought to resolve some of these issues so that, as a teacher, you may make use of the reflective feedback gained while teaching in school, and accordingly, you try to improve the Mathematics learning in our schools.

4.2 OBJECTIVES

After going through this unit, you will be able to:

- explain the social, mathematical and practical importance of Mathematics curriculum;
- identify the reasons for teaching Mathematics as an essential component of school curriculum;
- familiarise yourself with the vision of school Mathematics curriculum;
- conceptualise the concerns of teaching school Mathematics;
- discuss the underlying principles of development of Mathematics curriculum; and
- analyse various trends in the development of school Mathematics curriculum.

4.3 NEED AND IMPORTANCE OF MATHEMATICS IN SCHOOL CURRICULUM

Why do we need to know Mathematics? Why should we memorise so many formulae, theorems, proofs, etc? How will this information help us in our later life? What is its importance in my life? These are some of the common questions that we can see among those who are not interested in learning Mathematics. How far, as a teacher of Mathematics, we are able to convince our students to appreciate the importance of Mathematics?

'Why should we learn Mathematics?', is a valid question, and as Mathematics teachers, it is our responsibility to understand and conceptualise its importance and unique place among other school subjects. Why do our curriculum designers place Mathematics as a core school subject, and what is the significance of Mathematics in the overall school curriculum? The following values justify importance of mathematics curriculum.

4.3.1 Social Aspects

- The routine activities of daily life demand a mastery of number of facts and number of processes. To read with understanding much of the materials in newspapers requires considerable mathematical vocabulary. A few such terms are percent, discount, commission, dividend, invoice, profit and loss, wholesale and retail, taxation, etc. As civilization is becoming more complex, many terms from the electronic media and computers are being added.
- Mathematical operations like addition, subtraction, multiplication, division and so on, are used in our daily activities. From poor to rich, all have to use Mathematics in their real lives in one or the other way.
- Certain decisions require sufficient skill and understanding of quantitative relations. The ability to sense problems, to formulate them specifically and to solve them accurately requires systematic thinking.
- To understand many institutions and their management problem, a quantitative viewpoint (modeling) is necessary. It is illuminating to hear from an economist, an architect, an engineer, an aviator, or a scientist what in mathematics is helpful to them as workers.
- Many vocations need mathematical skills.

- The child should gain an appreciation of the role played by mathematics in many fields of work. Since, scientific knowledge and technology are linked with the progress and prosperity of a nation, we should be able to appreciate the role of mathematics in acquiring these.
- Mathematics has helped in bringing together the countries of the world which are separated from each other physically.
- Mathematics helped man to discover the mysteries of nature and to overcome superstitions and ignorance.

4.3.2 Mathematical Aspects

- Mathematics teaches us how to analyse a situation, how to come to a decision, to check thinking and its results, to perceive relationships, to concentrate, to be accurate and to be systematic in our work habits.
- Mathematics develops the ability to perform necessary computations with accuracy and reasonable speed. It also develops an understanding of the processes of measurement and of the skill needed in the use of instruments of precision.
- Mathematics develops the ability to
  - a) make dependable estimates and approximations,
  - b) devise and use formulae, rules of procedure and methods of making comparisons,
  - c) represent designs and spatial relations by drawings, and
  - d) arrange numerical data systematically and to interpret information in graphic or tabular form.

4.3.3 Application of Mathematics

- The history of mathematics is the story of the progress of civilizations and culture. “Mathematics is the mirror of civilization”.
- Egyptian and Babylonian civilizations have given a pertinent position to Mathematics. They considered it as a subject to be learnt in order to perform daily life activities in a better way. Elementary arithmetic and algebra were built up to solve the problems related to commerce and agriculture. They used this knowledge generally for money exchange, simple and compound interest, computing wages, measuring weights and lengths, determining areas of fields, etc. Since ancient times, the subject of Mathematics has been given a pivotal position due to its utilitarian and disciplinary values. It is believed that study of Mathematics improves our mental power and reasoning ability.
- A country’s civilization and culture is reflected in the knowledge of mathematics it possesses.
- Mathematics helps in the preservation, promotion and transmission of cultures.
- Various cultural arts like poetry, painting, drawing, and sculpture utilise mathematical knowledge.
- Mathematics has aesthetic or pleasure value. Concepts like symmetry, order, similarity, form and size form the basis of all work of art and beauty. All poetry and music utilizes mathematics. Quizzes, puzzles, and

magic squares are both entertaining and challenging to thought. Hence, the teaching of mathematics is inevitable in our schools.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

1) Mathematics is the mirror of civilisation. Do you agree with this statement? If yes, why? If no, why?

....

....

....

....

....

2) Discuss any two social aspects of mathematics in school curriculum.

....

....

....

....

....

4.4 VISION OF MATHEMATICS CURRICULUM AT SCHOOL LEVEL

In the previous section, we discussed how basic mathematical knowledge helps to develop the values, such as development of concentration, the power of expression, attitude of discovery, self reliance, economical living. That's why Mathematics is essential for learning other subjects. When we talk the vision of any subject, we need to consider all these aspects.

It is in this context, that National Curriculum Framework (NCF)-2005, stated that the **main goal of Mathematics education in schools is the mathematisation of the child's thought processes**. Basically, it means that children should learn to think about any situation using the language of Mathematics. Further NCF argued that, for the realisation of this vision, school Mathematics needs to recognize and try to work to achieve aims.

4.4.1 Aims and Objectives of Mathematics Curriculum

The narrower aim of teaching Mathematics at school is to develop useful capabilities, particularly those relating to numeracy- numbers, number operations, measurements, decimal and percentage.

The broader aim is to develop the child to think and reason mathematically, to pursue assumptions to their logical conclusions and to handle abstractions. School Mathematics curriculum should help the children learn to enjoy Mathematics. How can these visions materialise? The vision discussed earlier needs to be put to practice in the form of goal directed activities. The following objectives will help us in realising the vision of school Mathematics curriculum:

- ✓ Attain proficiency in fundamental mathematical skills;
- ✓ Comprehend basic mathematical concepts;
- ✓ Develop desirable attitudes to think, reason, analyse and articulate logically;
- ✓ Acquire efficiency in sound mathematical applications within Mathematics and in other subject areas;
- ✓ Attain confidence in making intelligent and independent interpretations; and
- ✓ Appreciate the power and beauty of Mathematics for its application in science, social sciences, humanities, arts, etc.

You may be remembering that we have already discussed in section 2.3 of Unit 2 about the aims and objectives of Mathematics in detail. So please refer it for further clarity.

4.4.2 Principles of Formulating Mathematics Curriculum

The curriculum is a tool to achieve the anticipated objectives of teaching a particular subject. **Curriculum can be considered as the sum total of all the experiences gained by a child as a result of various formal as well as informal activities at school, at home and in the society**. Curriculum is the formal and informal content and process by which child gains knowledge, develops skills, and modifies attitudes, appreciates the values.

What are the various components to be taken care of while designing a curriculum? As it is discussed that the curriculum transaction provides experiences to the students. What is the reason for that? One can say that, it is because of the vision of teaching a particular subject. If mathematisation of mind is our vision of teaching Mathematics in our school, we must provide appropriate activities to the students for attaining that vision. Again, we found that this vision needs to be materialised through various objectives. Hence, the **basic component while designing a curriculum is nothing but the pre determined objectives. The next aspect is the curricular activities to be provided to the students for realising these objectives.** The activities need to be planned for all content areas.

Thus, we have the objectives and the activities to be provided to the students. These two alone will not give us the desired result. How is the teacher going to transact or implement these activities in the classroom is also equally important. **The pedagogical approach that a teacher is going to use for organizing those activities will decide whether the child will be able to learn the concept or not. The next component is the student assessment.** This will help the teacher to understand whether the child has achieved these pre-determined objectives as a result of experiences he/she gained through the activities provided by the teacher. Thus, while constructing a curriculum; one should keep in mind those aspects. There are certain principles which help us to construct a better curriculum.

We know that the main aim of education is all round development of a child. In the earlier sections, we found that Mathematics plays a very crucial role in this process, since it is directly related to everyday life of children. **Hence, while**

constructing the Mathematics curriculum we need to consider those topics or themes, which would help children to succeed in their everyday life. The topics like interest, percentage, ratio, data interpretation, graphs etc are some of those topics.

Secondly child's needs, interests and capabilities should be considered as the base for curriculum construction. As the whole process of education is now going to be child-centred that means a curriculum must be child-centric. The curriculum must provide an environment conducive to learning where children feel secure, free from fear and compulsion. The curriculum must be helpful in developing an initiative, cooperation, healthy competition and social responsibilities among the children.

The content and various activities provided in the curriculum should help the students to understand the social and civic responsibilities. We are living in a society. Being a member of the society, the child should understand and learn about the society and contribute to the development of the society. For that purpose examples and activities, as far as possible, be related to the need of society.

Conservation of our cultural heritage is an important aspect that needs to be taken care of while framing curriculum. Education is regarded as a means for preserving our cultural heritage. Preservation and transmission of culture are two complementary components in this process. Mathematics curriculum also needs to follow this principle. Where ever possible, we need to include some activities, which would help in preserving and transmitting the cultural heritage.

Can we introduce the concept of proportion before introducing ratio? Can we teach simple and compound interest before teaching percentage? Can we teach algebra before teaching arithmetic? These questions are related and need to be addressed in a logical way. At the same time we need to consider the psychological development of the child also, while introducing a topic. Introducing algebra in class VI is not advisable due to the psychological and developmental stage of child.

For learning other subjects like physics, chemistry, economics, etc, the knowledge of Mathematics is essential. If we can relate and connect these with suitable examples from these subjects, the child will be able to appreciate the value of Mathematics.

The curriculum should be framed in such a way that different types of children can have opportunity for self-expression and development. Various activities need to be provided to the child according to the psychology of individual difference. In other words, the curriculum should be flexible in nature.

The development of mathematical concepts is not static. **The most modern and latest development in mathematical ideas should be included in the curriculum.** The topics which are not relevant to the present situations should be removed and new topics should be included in the syllabus. For example, the topics such logarithm may not be relevant at present but data analysis may be important and needs to be included in the curriculum. Hence, curriculum should be more dynamic in nature.

4.4.3 Core areas of Concern in School Mathematics

How far does our curriculum reflect the vision and objectives stated earlier? Whether the contents and materials given in the syllabus are appropriate to transact the vision into reality? These are some of the core issues to be examined carefully and addressed accordingly.

One of the major **areas of concern is that our curriculum is incapable in making Mathematics learning an enjoyable process.** What we see is that most of our children consider it as a difficult, boring and fearful subject.

On the one hand, it **develops a feeling of failure** among a large majority of students, and on the other hand, it **creates disappointment among a minority of gifted** or talented students. If we are not able to provide opportunity for these talented children to enjoy learning Mathematics, and to utilise their creative abilities effectively, slowly they would also start hating the subject.

Continuous and periodic assessment of students' learning is required to determine the extent to which the pre-determined objectives have been achieved. We may think that it is a very simple activity. But, an effective assessment requires ingenuity and innovativeness. **The theoretical knowledge provided to the teachers alone will not help them to implement effective assessment strategies in the classroom.** The teachers should be provided with various examples of assessment strategies with demonstration. Hence, in the curriculum there should be a scope for discussing about different assessment activities.

While discussing various areas of concern in Mathematics, we cannot ignore teacher, the very important concern. The pedagogy followed by the teacher is positively related to the success of students.

Consider the teaching of the identity $(a+b)^2 = a^2+2ab+b^2$. Let us see how this topic is introduced by the Teacher A and Teacher B.

Teacher A

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a(a+b)+b(a+b)\\ &= axa+axb+bxa+bxb\\ &= a^2+ ab+ba+b^2\\ &= a^2+ 2ab+b^2\end{aligned}$$

Teacher B

The teacher starts with discussion of the area of a rectangle and square by providing the students with various figures.

Tr : If side of a square is 3cm, then what will be the area?

St : 9 cm²

Tr : Shows a square piece of hardboard and asks one student to find out the area after measuring its side.

St : Side is of length 7cm. Hence the area will be 49 cm².

Tr : Shows another square of hard board and asks 'if the length of one side of this hardboard is 'a' cm, then what will be the area? [Fig. 14.1(i)]

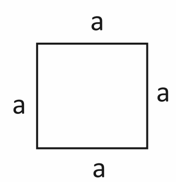


Fig. 14.1(i)

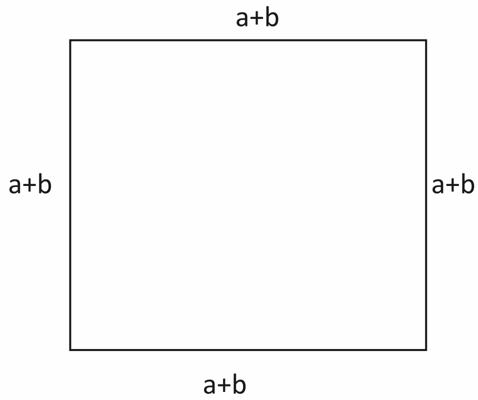


Fig. 14.1(ii)

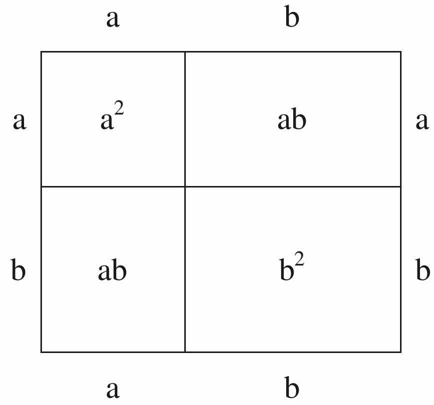


Fig. 14.1(iii)

St : a^2

Tr : If the length is a+b, what will be the area? [Fig. 14.1(ii)]

(Here draw a square with side 'a+b')

St : $(a+b)^2$

Tr : Can we express this in any other form?

Teacher marks the sides of the square. [Fig. 14.1(iii)]

The square hardboard has been divided into four parts, out of which two are squares and the other two are rectangles. Teacher guides the students to use different colours for denoting these parts. Students recognised that the area of the whole square is the sum of the areas of all the four parts. They also found that areas of the square parts are a^2 and b^2 respectively, and those of the rectangles are ab and ba respectively.

Hence, they concluded that the

$(a+b)^2 = a^2 + 2ab + b^2$ = bigger square + smaller square + one rectangle + other rectangle.

Just see the pedagogical approach used by the two teachers. Which teacher will be successful in their venture? There will be no doubt that teacher B will be more successful than teacher A. Students learning through the activity provided by teacher A will only learn through memorisation. On the other hand, students learning through the activity provided by teacher B, will be more opt to comprehend the idea properly. The same topic can be transacted in different ways. **The pedagogical approach used by the teacher has a crucial role in making the subject of Mathematics easy or difficult to the students**. **Proper orientation and training needs to be provided to Mathematics teachers in the area of pedagogy and practice**. In pre-service and in-service teacher education programmes, this concern needs to be properly addressed.

4.4.4 Curricular Choices at Different Stages of School Mathematics

What are the different content areas or themes to be included at various levels of school education? Do we include all topics at all levels? These are some of the issues to be addressed while constructing the curriculum. The choice of the topics should be in tune with the vision and accompanying objectives. While selecting a particular topic or theme, its suitability to the students of that level should also be taken care of.

Curricular Choices at Primary Stage of School Mathematics

Primary school stage is the basic platform on which the later stages need to be built. Two things have to be considered here while choosing the content. Firstly, this is the stage at which we need to create among children a positive attitude towards the subject of Mathematics. Naturally, young children like to play games and indulge in activities. Hence, ample scope must be given to include more Mathematical games, puzzles and other recreational activities at this stage. Secondly, since this is the level at which basic concepts of various mathematical ideas are to be developed. Higher mathematical concepts can be developed only if, children have a strong base. Hence more efforts needs to be made for creating strong base for Mathematics through effectively organising various activities. Most of these concepts have direct implications for day to day life of the child. Hence, as far as possible these contents should be taught by connecting them with everyday life situations. Various topics to be included at this stage may be number and its operations, shapes, spatial understanding, patterns, measurement and data handling. The curriculum must explicitly incorporate the progression that learners make from the concrete to abstract, while acquiring concepts. Apart from computational skills, stress must be laid on identifying, expressing and explaining patterns, on estimation and approximation in solving problems, on making connections, and on the development of skills of language in communicating and learning.

Curricular Choices at Upper Primary Stage of School Mathematics

During Upper primary stage, slowly children must be given opportunity to deal with abstract concepts. In order to sustain their interest and make them learn mathematics without fear and boredom, care must be taken to provide various mathematical games, puzzles, shortcuts, and recreational activities. This is the stage at which algebra needs to be introduced. It should be introduced by connecting it to real life situations and through its use in solving various life problems. This is the stage at which the concepts like percentage, ratio, proportions, interest, etc. are to be introduced. Mere introduction of these topics without connecting them with real life situation is of no use. The systematic study of space and shape and for consolidating their knowledge of measurement, data handling, representations, and interpretation form a significant part of the curriculum at this stage.

Curricular Choices at Secondary Stage of School Mathematics

At the secondary stage, students begin to perceive the structure of Mathematics, as a discipline. Pure rote-learning until facts are memorised mechanically, is to be avoided and opportunities to be provided to relate conceptual knowledge accompanied with procedural knowledge. Examples and activities to be provided for familiarising various concepts through the characteristics of mathematical communication, carefully defined terms and concepts, the use of symbols to represent them, and precisely stated propositions and proofs justifying propositions. Students develop their familiarity with algebra, Mathematical modelling, data analysis and interpretation during this stage. Algebra and arithmetic can be correlated with geometry. Algebra and geometry can be correlated with trigonometry. Attention must be paid also to the relationship of Mathematics with other subjects such as physics, chemistry, biology, geography or social sciences.

Curricular Choices at Higher Secondary Stage of School Mathematics

Mathematics curriculum at the higher secondary stage should make the students realise a wide variety of mathematical applications and equip them with basic tools that enable these applications. The correlation with other subjects like physics, chemistry, biology, economics, commerce, astronomy, computer science, etc needs to be emphasised at this stage. This stage is the launching pad from which the student is guided towards career choices. By this time, the students' interests and aptitudes have been largely determined and Mathematics education in these two years can help in sharpening their abilities. Greater attention is to be given for preparing children to the subsequent study of Mathematics at higher levels.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

3) Vision of teaching Mathematics is mathematisation. What do you mean by mathematisation?

....

....

....

....

....

4) *To develop desirable attitudes, to think, reason, analyse and articulate logically* is an objective of teaching Mathematics at secondary level. Give an example from secondary school Mathematics to explain how you will arrange learning activities to attain this objective.

....

....

....

....

5) Student assessment is a curricular concern. Can you illustrate this statement with the help of present system of assessment practices?

....

....

....

....

6) What do you mean by flexibility in curriculum? Give examples from secondary school Mathematics.

....

....

....

....

....

4.5 RECENT TRENDS OF CURRICULUM DEVELOPMENT

The paradigm shift from 'behaviourism' to the 'constructivism' as advocated at by NCF-2005, has to be seen as the recent trend in teaching-learning process at school in our country. This trend or shift has been the outcome of criticism, doubts, and questions about the validity of subject-centred approach to curriculum development, dominated by textbook contents that existed that time. It is imperative that teachers should have a clear understanding of the various approaches and the trends in the development of curriculum. As we have already discussed the fact as to how the value and importance of Mathematics will influence the development of school Mathematics curriculum, the understanding of different concepts of Mathematics is as important to the development and successful implementation of programs in school Mathematics. The trends in curriculum development need to be understood based on these aspects.

There are many approaches to curriculum development. Various approaches have been developed on the basis of theoretical perception of the curriculum developers. **The four major components in any educational process are curriculum, teachers, students and the contexts**. The various theories of curriculum development have interpreted these four essential components in different ways. Some curriculum developers focus on students and their learning goals, whereas others focus on the effects of the teacher's action upon learning and their pedagogical approaches.

4.5.1 Subject-Centred Approach

We have been following subject centred curriculum for a long time. This is also known as 'Traditional Curriculum' and we have moved away from this after the implementation of NCF-2005. This approach to **curriculum lays more emphasis on content in comparison to learners and teaching process**. The needs and interests of the child has no place in this curriculum. Here, **teachers' role is very crucial who are expected to transact the curriculum with a view to help students to learn different subjects.** A child has to learn the subjects in a time bound manner. They are expected to learn and memorise facts, concepts, processes, skills etc, and reproduce them at the time of examination.

4.5.2 Behaviourist Approach

According to behaviourists learning is nothing but modification of behaviour as a result of experience. In this approach, the **development of curriculum starts with a plan, called blueprint. Blue print consists of goals and objectives of learning of the particular subject.** The topics, contents and activities are to be planned on the basis of these pre-determined objectives. The duty of the teacher is to provide for the activities specified for realising these objectives. The student assessment, basically in the form of written knowledge and tests, needs to be conducted to know how far these objectives have been achieved. This approach suggests that **teacher should disseminate information in a sequential way** and demonstrate how to solve a problem, how to derive a formula, and how to construct a shape, followed by independent practice by students.

For example, teacher defines 'median' and demonstrates the derivation of formula for its calculation. This is followed by demonstration of solving a problem by the teacher, and another problem by students independently.

The role of students in this approach is to repeat what teacher transacted in the classroom. They are expected to listen, while teacher explains the content, demonstrates the processes. They should improve their learning through repetition or practice and reproduce the same when and where required.

4.5.3 Constructivist Approach

Constructivism believes that each and every child has the capacity to construct their own understanding and knowledge about various things through experiencing and reflection. **It is based on the premise that whenever a child encounters a new experience, he/she can either easily connect it with the existing knowledge or can make some changes in the existing knowledge to accommodate the new experience.** Piaget, a famous constructivist psychologist said that Mathematics is a subject, which may be very difficult to teach, instead, it has to be 'constructed' by the child. For this purpose, the child should be provided with various types of activities, which are essential for building mathematical shemas. Vygotsky, another psychologist, also known as social constructivist argued that it is as a result of the social interaction between the growing child and other members of the society that the child acquires the tools of thinking and learning. Constructivism holds that prior knowledge forms the foundation on which new learning is built. Because people and their experiences are different, they arrive at school with varying levels of proficiency. It is very clear that in constructivism major focus is on the learner and the learning process. Hence we have two variations in constructivist approach of curriculum development. These approaches are discussed below.

4.5.3.1 Learner-Centred Curriculum

In this approach, **the needs and interest of learners are paramount.** Those facts, concepts, theorems, processes, skills, etc, which are very essential for the child, should have a place in the curriculum. While discussing the value of teaching Mathematics, we found its applicability in various areas. Learner centred curriculum will help to inculcate these values by incorporating appropriate contents and activities. Here the role of **student will be that of an active participant in the learning process, and therefore, it necessitates that the teacher should know well each child** . Teachers try to unfold what each student already knows and inform how that knowledge can be utilised for further learning.

Learner centred curriculum will definitely **help the child to enjoy Mathematics, to make him realize its beauty, and to remove the fear of difficulty of the subject**. Another benefit of this curriculum is its flexibility. The new development and thinking in the area of Mathematics can be included at any time through the modification of the curriculum.

4.5.3.2 Activity-Centred Curriculum

This is also very similar to learner centred curriculum. The role of the learner is very important and should be very active. This is based on the **premise that child loves to play and activity will help to create motivation.** When curricular material is presented in terms of activity, it is known as activity-centred curriculum. **Learning of the prescribed material included in the**

curriculum takes place through appropriate activities. Another benefit of this approach is that throughout the teaching period, the students should be active participants in the process of learning. A goal directed activity should end in productive experience. In activity-centred curriculum, the content is presented through activities and knowledge is the outcome of these activities in terms of experiences.

Check Your Progress

Note: a) Write your answers in the space given below.

b) Compare your answers with those given at the end of the Unit.

7) What are the similarities and differences between subject centred curriculum and Behaviourist curriculum?

....

....

....

....

....

8) Differentiate between Learner centred curriculum and Activity centred curriculum.

....

....

....

....

....

4.6 LET US SUM UP

In this unit we have discussed that :

- Learning of Mathematics helps in daily life of an individual.
- Mathematics should be considered as a core subject area in school curriculum due to its wide applications.
- As per National Curriculum Framework (2005), mathematisation of the mind of the child will be the vision of school Mathematics.
- The narrower aim of teaching school Mathematics is to develop useful capabilities, particularly those relating to numeracy- numbers, its operations, measurements, decimals and percentages.
- The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusions and to handle abstractions.
- In order to fulfil the vision of mathematisation, curriculum should be constructed based on measurable and attainable objectives.
- While developing Mathematics curriculum with a vision of mathematisation, curriculum designers should keep in mind some basic principles of curriculum development.

- Some of the curricular concerns like teacher preparation, pedagogical-approaches, student assessment, etc, needs to be taken care of, while developing the curriculum.
- Appropriate topics need to be included in Mathematics curriculum at various stages of school education.
- Shift from subject-centred curriculum to constructivist approaches like learner-centred and activity-centred curriculum and their benefits.

4.7 UNIT END EXERCISES

1) What are the two aims of Mathematics education as per guidelines of NCF-2005? Explain in reference of nature of Mathematics.
2) In what sense should Mathematics curriculum be : ambitious, coherent, important?
3) State and explain different principle of construction of curriculum. Illustrate in reference of Mathematics.
4) What the core concerns of Mathematics education are as stated in N CF-2005? Do you agree with them? If yes, justify your answer.
5) Construct syllabi of statistics and probability for secondary level. Explain the advantage of keeping a constructivist approach in selecting topics for different stage of schooling.
6) 'Mathematics is a sequenced subject'. Illustrate the statement with the help of examples.

4.8 ANSWERS TO CHECK YOUR PROGRESS

1) Yes, Mathematics took its form, structure and language from the needs of human being. The history of mathematics has proved it. It has widespread application in various walks of our lives starting from home to universe. The statement illustrates the utilitarian value of learning mathematics.
2) Refer section 4.3.1.
3) Mathematisation is the mental process through which mathematics can be developed. Simply rote memorising a formula and solving a problem will not produce mathematics. Hence this cannot be equated as mathematisation. This requires intelligent use of various mathematical processes.
4) Consider the application of trigonometry in everyday life. Take any example of puzzles related with height and distance problem. How do we measure the height of a tree or any other object without using measuring too. Ask the students to discuss in groups. The various process involved in the problem solving situation definitely will be useful for realizing the mentioned objective.
5) National Curriculum Framework 2005 talks about the importance of integrating assessment with teaching learning process. Consequently Continuous and Comprehensive Evaluation has been implemented in school education system. The basic tenet of this assessment is that, it should be integrated with classroom activities. Teacher needs to plan in advance various strategies needs to be used for assessing student

performance continuously. In this way it can be considered as a curricular concern.

6) Flexibility of curriculum means, it should include activities for the requirements of different types of learners. At the same time activities provided should not be prescriptive in nature rather it should be suggestive in nature. The teacher may be given freedom to organize appropriate activities. For example while teaching statistics, rural students may be provided examples from their local environments while urban students examples may be different from those.

7) Subject centered curriculum centered on a single subject with a pre determined standard. Learning that subject is an end itself and may not be linked to other subject. Here the role of teacher is very crucial and student has no freedom in managing the classroom process. Behaviourist curriculum also resembles like subject centered curriculum. In behaviourist approach the whole curriculum will be pre planned based on a blue print.

8) Both learner centered and activity based approaches are based on the principle of child centeredness. Both ensure active participation of children in teaching learning process. Another important feature of these approaches is the flexibility of curriculum. Teacher needs to provide appropriate activity to students based on their requirements. Both will help to enhance construction of knowledge.

4.9 REFERENCES AND SUGGESTED READINGS

- Anthony, G. and Walshaw, M. (2009). Characteristics of Effective Teaching of Mathematics: view from the West, *Journal of Mathematics Education,* Vol 2, No.2, pp147-164
- Beckmann,C.E,; Thompson, D.R. and Rubenstein, R. N. (2010). *Teaching and Learning High School Mathematics,* John Wiley and Sons Inc. New Jersey
- Chambers Paul (2010). *Teaching Mathematics- Developing as a Reflective Secondary Teacher*, Sage South Asia Ed, New Delhi
- Hansen Alice and Vaukins Diane (2012). *Primary Mathematics across the Curriculum,* Sage Publication India Pvt-Ltd, New Delhi
- National Council of Teachers of Mathematics (2000). *Principles and Standards for school Mathematics*. Reston V A, NCTM
- NCERT(2019). *Textbooks for Classes IX and X*, NCERT, New Delhi
- NCERT (2005). *National Curriculum Framework, 2005,* NCERT, New Delhi
- NCERT (2005). *Position Paper National Focus Group on Teaching of Mathematics*, NCERT, New Delhi
- NCERT (2012). Pedagogy of Mathematics, Textbook for Two Year B.Ed Course, NCERT, New Delhi
- Spangler, David B (2011). *Strategies for Teaching Fractions*, Sage Publication India Pvt-Ltd, New Delhi